Time Domain Response Surface Model Updating for Nonlinear Structures

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ABSTRACT

Finite element (FE) model updating aims to modify the uncertain parameters of a FE model to improve the correlation between certain analytical response features and their experimental counterparts. One of the proposed methods to solve this optimization problem is to approximate the input-output relationship of deterministic FE models with polynomial functions and estimate the uncertain parameters of the model using these approximated surrogate models, called Response Surface (RS) models. In this study, low computational effort associated with RS modeling is used to extend nonlinear FE model updating through time domain data. For this purpose, with assumption of known input and using least square techniques, RS models are constructed at every time step of the analysis and minimization problem of parameter estimation is solved iteratively in length of the time history of responses of the system. This paper investigates sensitivity of such RS estimates to measurement noise and input frequency in FE model updating of nonlinear structures. Further application of this procedure is investigated in a case study of a steel frame with bilinear material model under seismic loading.

INTRODUCTION

Structural Health Monitoring (SHM) process covers a group of procedures involved in instrumentation, response measurement, and evaluation of the structures' state of health. The main elements of SHM in data interpretation fall into three groups: Identification of modal characteristics of the structures [1, 2]; damage detection and localization algorithms [3]; and techniques to calibrate FE simulations based on measured responses of the monitored structures.

While these techniques share a common objective in assistance with lifetime maintenance of the structures, experimentally validated FE models proves to be beneficial in other applications such as reliability study under various conditions, assessment of retrofit alternatives, improving efficiency of model-based damage detection methods, etc.

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FE model updating is a constraint optimization problem in which uncertain model parameters are modified to minimize the discrepancy between certain measured response features and their analytical counterparts. Selection of such response features mainly depends on the behavior of the structure and the future application of the calibrated model. When the FE model is updated to represent the structure behaving linearly in low levels of vibration, experimentally identified modal quantities (natural frequencies, and mode shapes) are commonly used for estimation of the model parameters. However, when a system behaves nonlinearly, such features fail to estimate model parameters and other metrics are required for FE model calibration.

Several procedures have been proposed to reduce the uncertainty associated with the parameters of the linear FE models. These procedures mainly work based on sensitivity analysis and iterative calculation of the local gradients of the selected responses as functions of updating parameters [4]. In order to overcome intensive computations and possible convergence problems associated with these procedures, application of RS methodology was introduced in the process of FE model updating [5, 6]. RS models are mathematical expressions that approximate the relation between pre-selected inputs and output of the FE model. Polynomial functions are commonly used for this purpose. These RS models are used as surrogate to replace the FE model in the optimization problem of model updating. Previous applications of this method in updating the uncertain parameters of linear FE models proved the success and efficiency of this method over traditional sensitivity based approaches [7, 8].

In this study, RS models are used to update nonlinear FE models in time through a procedure previously proposed by the authors [9]. This technique consists of three steps of RS model construction, evaluation, and optimization. Through these three steps, with assumption of known input and using least square techniques, accurate RS models are constructed at every time step of the analysis and minimization problem of parameter estimation is solved iteratively in the length of the time history of responses of the system. Figure 1 presents the flowchart of this methodology.

This paper investigates the sensitivity of the estimates of this technique to measurement noise level and frequency of input excitation. Further application of this method in updating a nonlinear model under seismic load is also investigated.

SENSITIVITY OF THE RS ESTIMATES TO NOISE AND LOADING FREQUENCY

This section investigates the sensitivity of RS estimates to measurement noise and loading frequency. For this purpose, a single degree of freedom system was simulated with unit mass and bilinear stiffness material model under harmonic loading. Initial stiffness of the system (k) and yielding force are 5 lb/in and 5 lb, respectively. Post yielding stiffness ratio of the system (α) was selected as uncertain model parameter varying between 0.2 and 0.8. Time history of displacement of the mass is used to estimate α in this range. In different scenarios, frequency of the applied load varies from 0.1 to 10 Hz while the amplitude of load was adjusted so that in all the cases maximum displacement of the system lies in the same range (approximately 3.1 in).

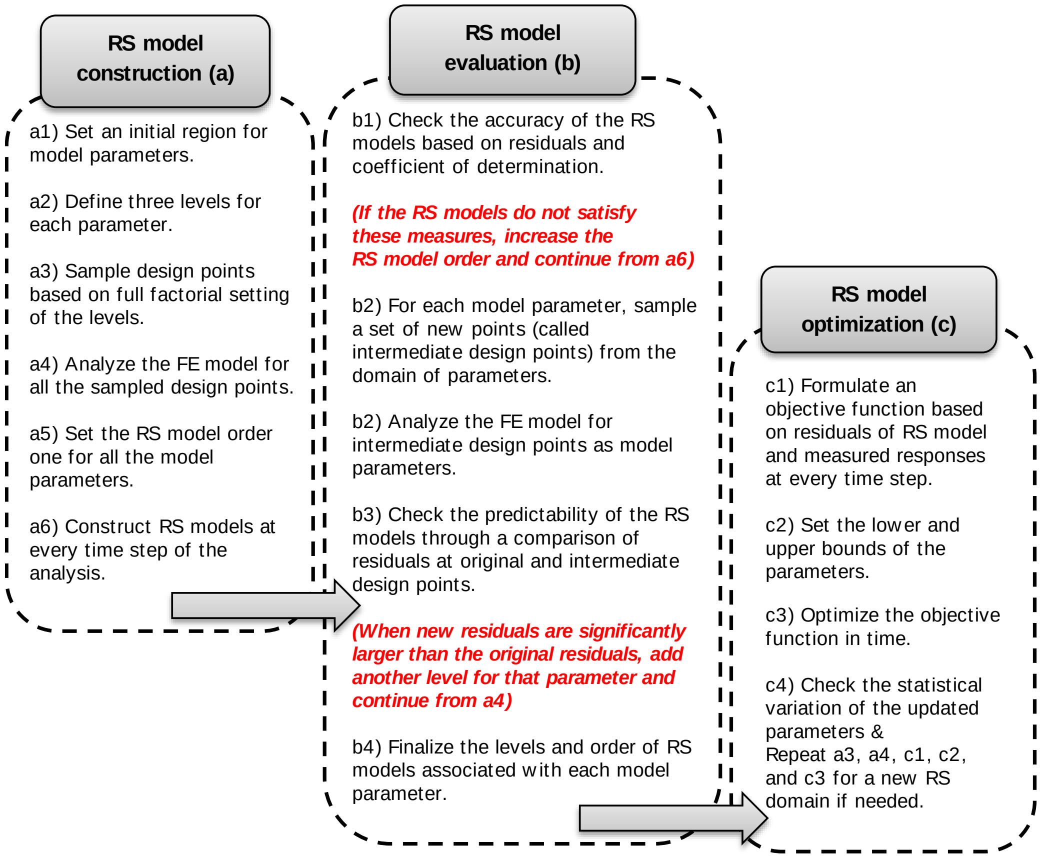

Figure 1. Flowchart of RS model updating in time domain

With sampling frequency of 1 kHz, a window with one second length was used to update α in every scenario. The model construction and evaluation steps resulted in three levels and quadratic RS models of displacement as function of α in the selected time window. Residuals of simulated displacement and regressed RS models were minimized along the selected time window to update α. Different levels of the measurement noise were assumed in each case. Noise level denotes the ratio of the root mean square of the simulated Gaussian noise signal to root mean square of the simulated measured signal. Figures 2 to 5 show the results of the updating procedures where α is set equal to 0.65 and 0.2 to simulate the measured displacement signal.

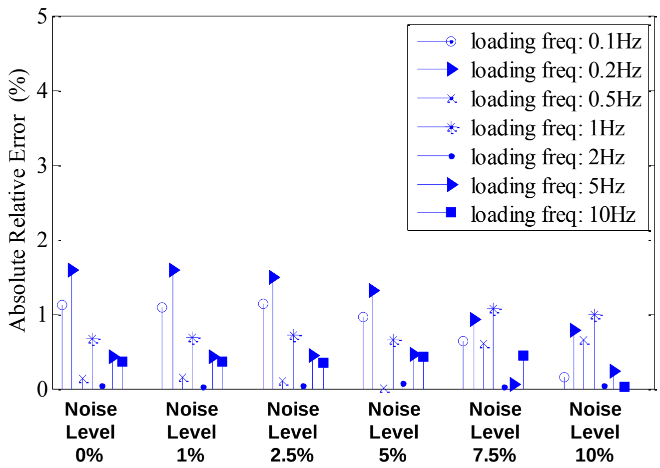

Figure 2. Error in mean updated α (α_{true}=0.65)

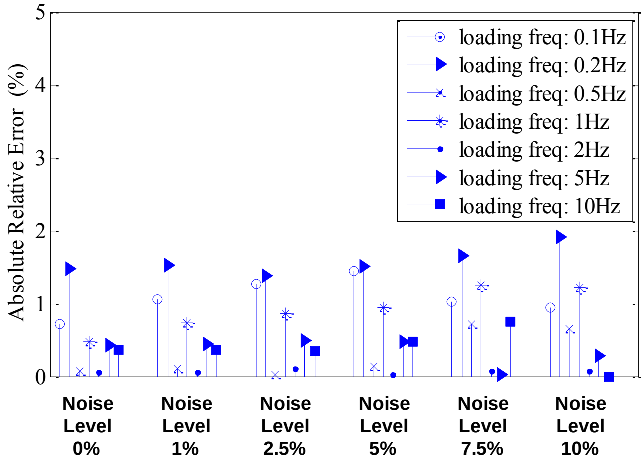

Figure 3. Error in median updated α (α_{true}=0.65)

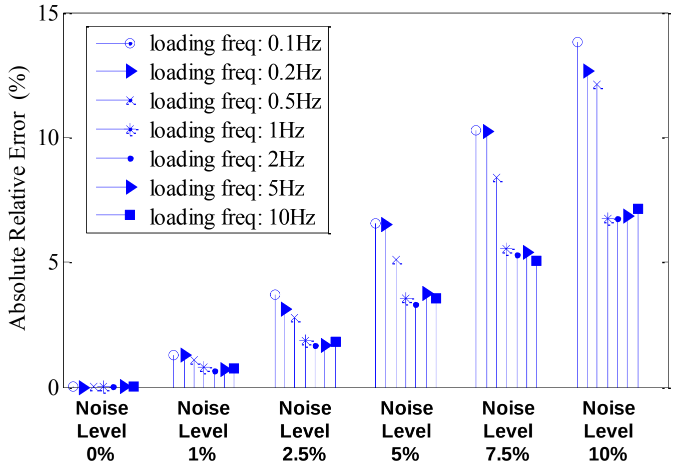

Figure 4. Error in mean updated α (α_{true}=0.2)

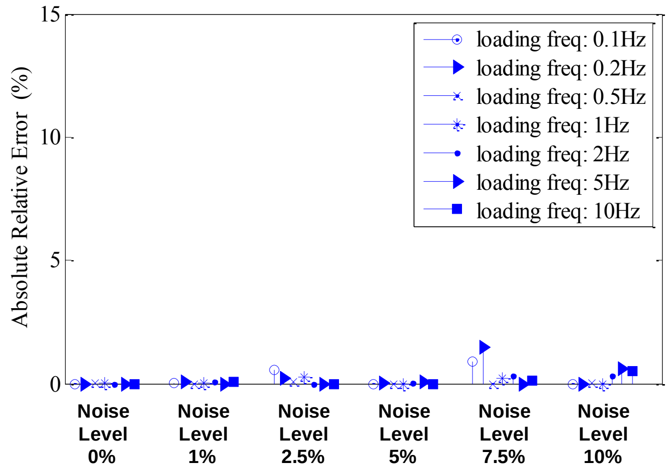

Figure5. Error in median updated α (α_{true}=0.2)

The results indicate that the proposed procedure successfully estimates α with different assumptions of measurement noise level and loading frequency.

Moreover, median value of the histograms of the updated α appear to be a more accurate point estimate of the updated α.

NONLINEAR FRAME SUBJECTED TO EARTHQUAKE LOAD

The previous section demonstrated that RS estimates are robust with respect to the frequency of the loading and measurement noise. This implies further application of this method in updating parameters of nonlinear models in time under seismic loading. To validate such application, in this section a steel frame with bilinear material model is studied. The frame consists of one span with overall length of 7’ 6” supported by columns 2’ 9” long. The cross section of the beam and column members is uniform hollow 2-inch tube, with 0.083 in wall thickness. The steel has bilinear material model with 50 ksi yield stress. Modulus of elasticity (E) and post yielding stiffness ratio (b) of steel are the updating parameters. To simulate the experimental data, E and b were set to 31500 ksi and 0.125, respectively, and a dynamic loading resulting from the EL Centro earthquake record was applied to the left column-beam joint. To update the pre-selected parameters of the model, simulated time histories of displacement at two locations on the frame were used. Figure 6 shows the configuration of the frame, loading and responses used for updating the FE model.

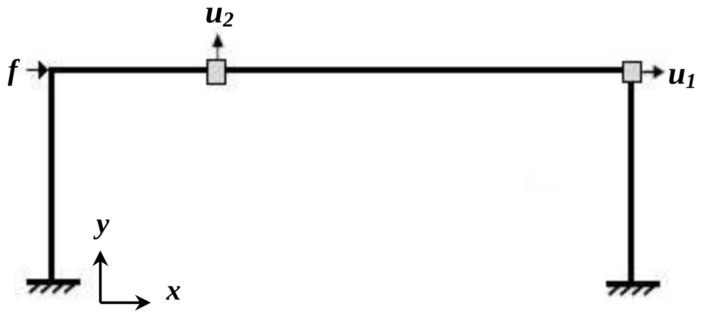

Figure 6. Configuration of the nonlinear steel frame

A 2-D massless FE model of the frame was developed in Opensees [10] using fiber section procedure and Steel01 uniaxialMaterial properties. Two cases of RS domain for model parameters are studied. In the first case, b and E were selected from [0.05 to 0.25], and [27000 to 33000 ksi] intervals, respectively. In the second case initial domain of b and E are [0.125 to 0.325], and [25500 to 31500 ksi], correspondingly.

The steps of model construction and evaluation in the first case resulted in 5x3 full factorial design of model parameters and model order of 4 for b and 2 for E. In the second scenario, these steps lead to a 3x3 design and quadratic RS models. In the optimization step, an objective function based on measured displacements and regressed RS models was iteratively optimized at every 0.004 second in a three-second long window associated with the response of system to the strong motion segment of the earthquake loading. Optimization step completed based on a multi-start framework using interior-point algorithm [11].

Figure 7 and 8 show the histograms of the updated model parameters in these two scenarios.

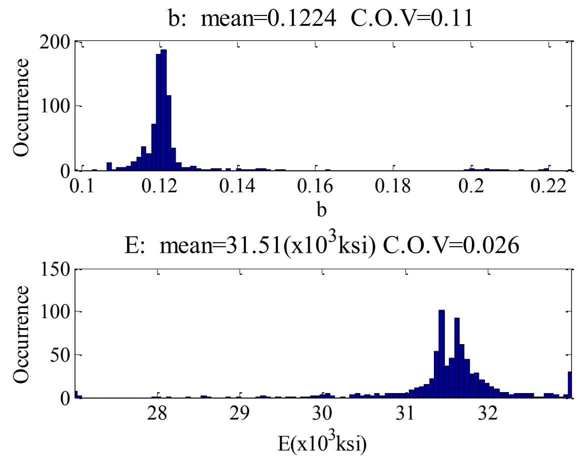

Figure 7. Histograms of the updated model parameters: Case 1 (Noise free data)

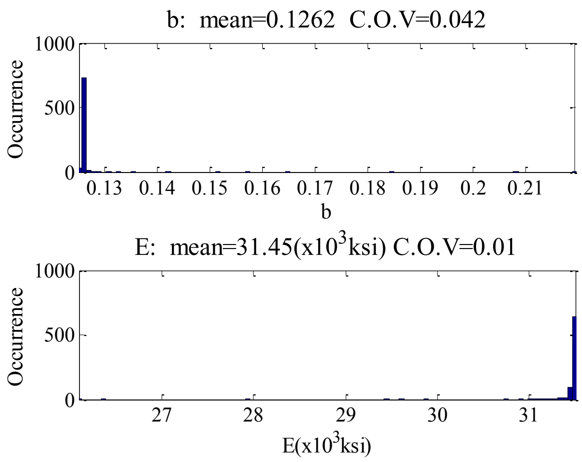

Figure 8. Histograms of the updated model parameters: Case 2 (Noise free data)

To account for the measurement noise, different levels of Gaussian noise were added to the simulated measured responses and optimization step iterated. Table I and II summarize the relative error associated with mean and median values of the updated parameters in all the scenarios.

TABLE I. RESULTS OF THE UPDATING PROCEDURE UNDER SEISMIC LOADING: CASE 1

Noise Level	Normalized Error (%)			
	mean b_{upd}	mean E_{upd}	median b_{upd}	median E_{upd}
0	-2.077	0.032	-3.461	0.294
1	-1.795	-0.033	-3.663	0.281
5	1.397	-1.368	-2.655	-0.187
10	4.851	-1.832	-1.698	-0.058
20	11.121	-2.693	1.577	-0.120

TABLE II. RESULTS OF THE UPDATING PROCEDURE UNDER SEISMIC LOADING: CASE 2

Noise Level	Normalized Error (%)			
	mean b_{upd}	mean E_{upd}	median b_{upd}	median E_{upd}
0	0.960	-0.159	0.542	-0.027
1	4.246	-1.373	0.915	-0.137
5	9.628	-3.425	2.826	-0.940
10	17.984	-4.802	5.966	-1.381
20	30.833	-6.091	12.528	-2.322

Tables I and II show that the median of the histograms of the updated model parameters more accurately estimate the true model parameters. This is the case particularly when true model parameters are located close to the corners of RS domain, as the histogram of the updated model parameters are highly skewed in these situations.

CONCLUSIONS

This paper investigate the sensitivity of a previously proposed nonlinear FE model updating technique under different assumptions of the measurement noise and loading frequency through a numerical case study of a single degree of freedom nonlinear system. The results indicate that the procedure is robust to these parameters. Further application of this procedure was also presented in a case study of a steel frame with bilinear material model under seismic loading.

ACKNOWLEDGEMENTS

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