The Use of Localized Sensor Networks for Damage Detection: A Likelihood Ratio Test Control Chart

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ABSTRACT

This study presents the implementation of an output only damage detection method on an experimental structure by applying a mulitvariate change point analysis approach. The multivariate method, the Normalized Likelihood Ratio Test, is utilized to create control charts in order to localize damage on a small scale steel girder. Data from a dense sensor network is utilized to correlate a location on the frame with damage.

1. INTRODUCTION

Structural health monitoring (SHM) research has become a vital tool in maintaining the integrity of structures that has been refined over the years. Literature shows numerous methods and techniques for damage detection from the past. The tests were not always easy to conduct because the location of the damage, the stiffness, and other properties of the material had to be known before damage detection methods could be implemented [1]. Additionally, areas of damage were not always easily accessible for instrumentation and the tests could only account for global damage within the structure. Many approaches used modal parameter identification with data in time and frequency domain, such as [2], in which stiffness estimation/damage detection is investigated in a shear frame structure. However, many types of these methods require high signal to noise ratio measurement data as well as moderate damage levels and are computationally intensive [3]. Research in improving these methods is critical to the advancement of the civil engineering field itself and to sustaining the infrastructure of today's buildings.

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As an answer to the shortcomings of these damage detection techniques, model-free approaches can be used. These methods use data from localized sensor networks to correlate acceleration with location. The data is implemented into an output only algorithm so that there is no need for a priori knowledge of the structure's properties or suspected location of damage. Sensors networks have been used to localize damage on various types of structures such as large-scale steel beam-to-column moment connection, [4], large-scale moment connections [5], and long-span bridges [6]. These approaches are easy to implement and effective in reflecting the changes in the structure's behavior; yet they are dependent on statistical analysis to determine the significance of the changes. One type of statistical approach is the use of control charts in which thresholds are used to distinguish between assignable and common changes in the data, a distinction that is extremely vital in damage detection.

Literature presents different statistical approaches which are developed for detection of changes in observations for different applications and disciplines including structural damage detection [7]. This study implements an output only damage detection method on an experimental structure using the Likelihood Ratio Test and the Moving Range control statistic. There are many different types of likelihood related tests that can be done and are presented in references [8]-[9]. For this implementation, a normalized likelihood ratio test is used and the response of a scaled steel frame is recorded from two states in order to detect damage. The first is a baseline healthy state of the structure, and the second is an unknown state. In effect, two sets of acceleration data are created that would be taken from a structure pre- and post- a damaging event. The control chart is used to make the distinction, if any, between an undamaged and an unknown state of the structure.

2. AN INTRODUCTION TO CHANGE POINT ANALYSIS STATISTICS

Control statistics can be used to monitor a change in a process. With the use of a threshold value, the significance of a change in a process can be determined. It is important in several industrial applications, as well as civil engineering, to know if a change in a process is due to slight changes over time (from common causes) and the process is still in control, or if the process is out of control due to a damaging event (or assignable cause). Control charts utilize a threshold to distinguish between the two. Once the threshold is crossed, the process can be deemed out of control.

2.1 Background

There are several different types of control statistics that can be used for change point detection in different processes. These processes can be used to detect a single change or multiple changes in the mean or variance of the data [10]. One of the first charts generated, the standard univariate Shewhart $\bar{X}$ control chart, was introduced in 1924 by Walter Shewhart. Since then, control schemes have found widespread application in different disciplines.

One major flaw in using univariate control statistics is that they can only monitor one variable at a time. If one were to observe a set of quality characteristics that have components with the potential to be interrelated, the univariate control schemes become obsolete. Although it could be argued that univariate control charts could be applied independently to each component of the multivariate data, misleading results may be obtained in some cases due to failure to allow for the inherent relationship among the components of the multivariate data [11]. Therefore, this paper explores a multivariate control statistic.

2.2 The Normalized Likelihood Ratio Test

The likelihood ratio test can detect a shift in the mean and variance of a data set. It assumes that there are *m* independent observations that are normally distributed with mean μ and standard deviation σ. If a process was in control, at any partition of the data, the two sets would have the same mean and variance. However, if there was a change in the process, the mean and variance would change. The objective of using a control chart is to find the significance of this difference by creating a threshold for comparison.

The log of the likelihood function for the first m_1 observations

$$l = \frac{-m_1}{2} log[2\pi\sigma^2] - \frac{m_1 \widehat{\sigma_1}^2}{2\sigma^2} log[\widehat{\sigma_1}^2] - \frac{m_1(\overline{x_1}-\mu)^2}{2} \quad (1)$$

is maximized for $\overline{x_1}$ and $\widehat{\sigma_1}^2$, the mean and variance of the first m_1 observations, to generate l_1 presented below.

$$l_1 = \frac{-m_1}{2} log[2\pi] - \frac{m_1}{2} log[\widehat{\sigma_1}^2] + \frac{m_1}{2} \quad (2)$$

This procedure can be repeated for the remaining m_2 $(= m - m_1)$ observations to create the maximum likelihood function, l_2. This formulation assumes that there is a change in the data at m_1. However, if the process was in control for all *m* observations the likelihood function would be maximized using $\bar{x}$ and $\widehat{\sigma^2}$, the mean and variance of the entire set of observations, to create l_o, the maximum of the likelihood function for an assumed in control process [12].

If l_a, which is the sum of l_1 and l_2, is much larger than l_o, this is an indication that the process is out of control. The likelihood ratio test detects the significance of the difference between the two. It is defined as $lrt[m_1, m_2] = -2(l_o - l_a)$ and has an asymptotic chi-squared distribution (χ^2) with two degrees of freedom.

This statistic is normalized to create the Nlrt with a threshold value of unity. Therefore, any value of the likelihood ratio for the damage features that is above one, represents data from a location that has become out of control. The statistic is normalized by its expected value and an upper control limit which is determined for a desired overall in-control false alarm probability *p*.

$$UCL = \frac{1}{1.7} F^{-1}[(1-p)^{\frac{1}{k^*}}] \quad (3)$$

Here k* is the best fit number of independent variables and F is the CDF of the χ^2 distribution.

3. LOCALIZED DAMAGE DETECTION METHOD

The damage features studied in this paper come from linear regression coefficients produced by an algorithm called Influenced-based Damage Detection Algorithm (IDDA) developed by Dorvash et al [13]. It is essential in structural damage detection that the features used in the model reflect a unique pattern to the state of the structure [14]. This algorithm correlates the response of a structure at various locations by creating influence coefficients from a linear regression model based on output of a dense sensor network. When damage occurs, the relationship between responses changes, which will be reflected in the influence coefficients and indicate the existence of damage. The location can then be pinpointed by correlating the location of the sensors.

3.1 The Mathematical Model

The relationship between responses at different locations of a structure can be established using an Auto Regressive with Exogenous term (ARX) model as:

$$\sum_{p=0}^{P} y_j(n-p) = \sum_{i=1}^{k} \sum_{q=0}^{Q} \alpha_{iq} y_i(n-q) + \varepsilon(n) \tag{4}$$

where y_j and y_i are outputs at locations j and i, respectively, α_{iq}'s are ARX coefficients, $\varepsilon(n)$ represents the residuals, n is the time index, and P and Q are orders of the autoregressive and exogenous parts of the ARX model, respectively. Derivation and validation of this formulation can be found in [15] and [16] respectively. Additionally, derivation and validation of a simplified mathematical model can be found in [17] on a small scale beam-column connection.

3.2 The Akaike's Information Criterion (AIC)

The accuracy of the auto regression model is dependent on the selected model orders. While higher model orders, in general, deliver more details of the system and reduce the estimation bias, it is always desired to keep the order at the minimum level to avoid over-parameterization. One way to establish the model order is to minimize the Akaike's Information Criterion (AIC) which is defined in [10] as,

$$AIC(k) = -2logL(\hat{\theta}_k) + 2k. \tag{5}$$

In Eq. 5, k is the number of parameters in the statistical model and $\hat{\theta}_k$ is the maximized value of the likelihood function. The AIC is calculated for the data set used in this paper and the model order was found to be four.

3.3 Mahalanobis Distances

As a result of having a set of influence coefficients from the auto regression, the α values need to be condensed in order to be used in the Nlrt statistic. The set of coefficients associated with a certain condition and a certain location on the structure can be used to create a Mahalanobis distance. This parameter generates the distance between selected damage features corresponding to a condition of interest and those corresponding to a reference condition.

The Mahalanobis distance $D_m(x)$ can be computed by using Eq. 6:

$$D_m(x) = \sqrt{(x-\mu)^T S^{-1}(x-\mu)} \tag{6}$$

Where μ is the mean of the vectors, S is the covariance and x is the damage indices matrix [18]. In detection schemes, a larger Mahalanobis distance indicates that the location is closer to damage. Now the data is ready to be used in the Nlrt control chart.

4. TEST SETUP AND IMPLEMENDATION OF IDDA

4.1 Test Specimen

A two-bay frame test bed was constructed at the laboratory of Advanced Technology for Large Structural Systems (ATLSS) at Lehigh University. In order to simulate damage, there are nine sections, $0.19\ m$ in length, that can be changed throughout the frame. These interchangeable sections have different cross sectional properties than the healthy state (shown in Table I) which correspond to a 20% reduction in member stiffness.

TABLE I: GEOMETRY OF BASELINE AND INTERCHANGEABLE SECTIONS

Feature	*Baseline Sections*	*Interchangeable "Damage" sections*
Outer Dimension of Hollow Cross Section	0.05m	0.05m
Tube Thickness	2.16 mm	1.65mm
Cross Sectional Area	410.57 mm^2	324.57 mm^2
Moment of Inertia	162526 mm^4	130811 mm^4

For this implementation, data was collected using 21 wired accelerometers (the use and validation of wireless sensors is investigated in [19]). Sensors were spaced throughout the two-span frame as shown in Fig. 1. During testing, there were 20 runs taken from the undamaged state of the frame which serve as a baseline for comparison. For this study, the damage case consists of replacing a healthy section with a section of less stiffness at the location corresponding to sensor R5. After the section is exchanged, an additional 20 tests were run. These tests will serve as the unknown state of the structure after a damaging event. For each run, the sampling rate was 500 Hz and 10,000 samples were recorded. Results from these tests are shown in section 5.

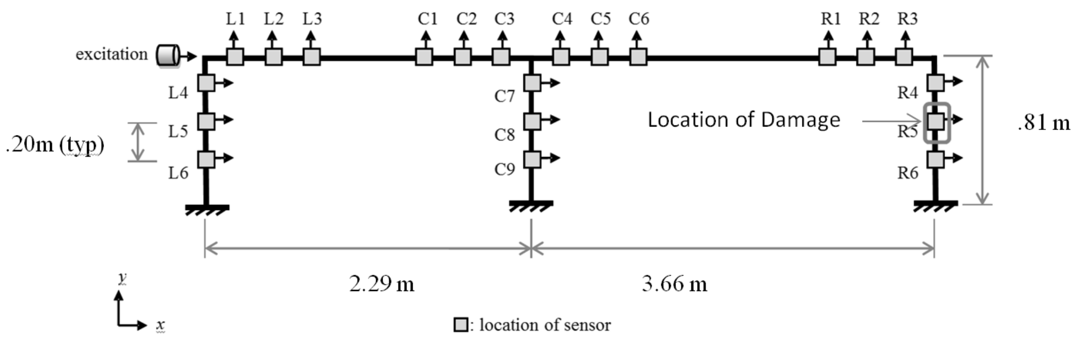


Figure 1: Sketch of the specimen and the location of the introduced damage.

4.2 Loading Scenarios for Implementation of Damage Detection Algorithm

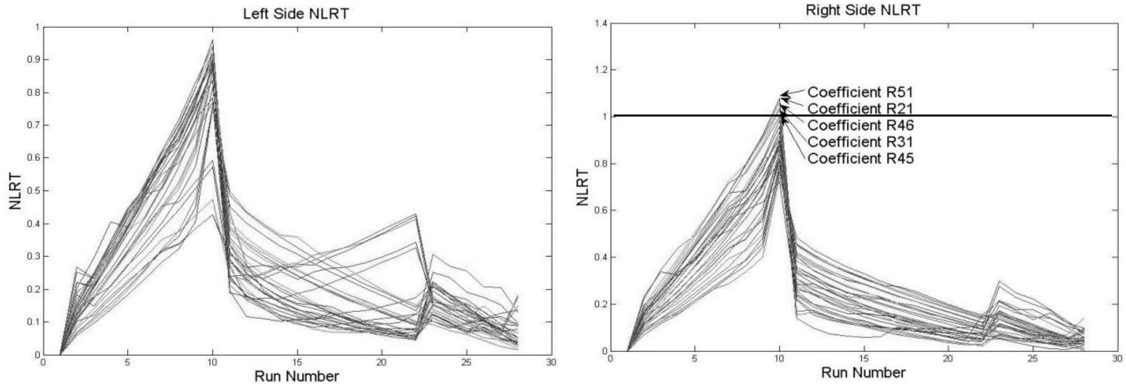
Data collection using the algorithm discussed would occur pre and post a catastrophic event or regular maintenance of a building. Therefore, it is possible to assume that the structure is subjected to ambient vibrations during data collections. Because this loading is generally categorized in the linear elastic range in which small angle theory applies, the frame can also be assumed to behave linear elastically [1]. Methods of excitation include random vibration, harmonic loading, and impact.

During impact loading the frame is struck with a hammer, the actuator is removed and the frame is free to vibrate on its own once it is excited. To simulate an impact test, the frame was hit with a hammer on the right column. The focus of this paper is kept to the impact test results.

5. Results and Conclusions

Because the damage case being considered is a section on the right side column, the results should detect damage at or near this location. Only the results on the left and right side of the girder are compared for simplification. There are six sensors on each side of the frame; consequently, thirty pairs can be made without comparing a sensor to itself. Multivariate damage features presented are calculated and analyzed for each sensor pair combination using the likelihood ratio test and the results are shown in the figures below.

From the figures, the damage features for ARX regression do show damaged pairings in the correct vicinity of the actual damage on the girder. Several coefficients cross the threshold on the right side of the frame, while none do on the left side.



(a) (b)

Figure 2.a and 2.b: Nlrt Results for the left and right sides respectively

Although there is no damage on the right side beam, these locations are close to the actual damaged section and therefore it is reasonable that these locations would experience more of a significant change in response. Since the correct location of damage is detected, the normalized likelihood ratio test using damage features from ARX regression can be used to detect damage in the right locations. It should be noted that more work should be done to localize these results even further. Additionally, different methods can be used to compare and correlate the findings.

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