Structural damage localization using sensor cluster based regression schemes

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ABSTRACT
Automatic damage identification from sensor measurements has long been a topic of interest in the civil engineering research community. A number of methods, including classical system identification and time series analysis techniques, have been proposed to detect the existence of damage in structures. Not many of them, though, are reported efficient for higher-level damage detection which concerns damage localization and severity assessment. In this paper, regression-based damage localization schemes are proposed and applied to signals generated from a simulated two-bay steel frame. These regression algorithms operates on substructural beam models, and uses the acceleration/strain responses at beam ends as input and the acceleration from an intermediate node as output. From the regression coefficients and residuals three damage identification features are extracted, and two change point analysis techniques are adopted to evaluate if a change of statistical significance occurred in the extracted feature sequences. For the four damage scenarios simulated, the algorithms identified the damage existence and partially succeeded in locating the damage. More accurate inferences on damage location are drawn by combining the results from different algorithms using a weighted voting scheme.

Keywords: damage localization; substructural analysis; vibration monitoring; regression modeling ;pattern recognition

INTRODUCTION

Damage detection and assessment is very important for timely and proper maintenance of civil infrastructures. Traditional practices rely a lot on human inspections, and as a result the cost of maintenance is high and the period of maintenance is long. With the development of digital sensing technologies, damage identification from sensor measurements analysis[1,2] has been proposed as an attractive alternative because of the low cost and easy scalability.

Macro-scale system identification/modal realization[3,4] is one of the earliest methods studied for structural diagnosis purposes, and a lot of literatures can be found that investigates the damage indication performance of the functions of eigenfrequencies and eigenvalues estimated from structural responses. While these damage features are theoretically well-grounded for understanding, it may take a lot of time and resources for some algorithms to achieve good results and the features are sometimes found to be insensitive to local damage. Also, model updating for the monitored system will often be needed to perform high-level damage detection. In an effort to overcome these problems, damage detection techniques that adopt time series analysis on single channel responses have been proposed [5,6]. These features are found to be sensitive to damage, but also to the excitation condition change as a result of the inherent information limitation for this family of methods. Besides, many of these established methods are found to not efficient for high-level damage detection such as damage location and extent identification.

In order to strike a balance between algorithm damage sensitivity and performance robustness, in this paper methods based on multivariate linear regression using measurements from sensor clusters are proposed. The underlying idea is a sub-structuring technique that models the response at a certain node as the output of a system with responses at all its neighboring nodes as input. Regression coefficients and functions of the regression residuals are used as damage indicators. It is shown that these features would still be sensitive to local damage, yet has better stability and damage localization capabilities. Change point analysis is used to construct the damage threshold for the all the features extracted, and the location of the features that exhibit the most significant change is recognized as the damage location.

This paper contains 6 sections. Section 2 contains description of two sub-structure models utilized for linear regression model construction. Section 3 introduces the formulation of damage features from regression models. Section 4 presents cumulative sum based and minimum deviation based change point analysis techniques for damage threshold determination. Section 5 summarizes the implementation results for all algorithms/damage indices, and their damage identification and localization performances are compared, contrasted and combined. In the end, conclusion is drawn on the merits and demerits of the methods used.

CONSTRUCTION OF SUBSTRUCTURAL BEAM MODELS

To apply the linear regression techniques for damage identification, we need to clarify an input-output linear model for the structure to be assessed. In this section, two models for beams are presented and associated input-output relations formulated.

The standard static beam model

It is known that the displacement $u(x)$ of a continuous beam with section stiffness $EI(x)$ when subjected to no intermediate load (Fig. 1) should satisfy the differential equation below:

$$\frac{d^2\left(EI(x)d^2u(x)\right)}{dx^4} = 0 \qquad (1)$$

This is a fourth order differential equation, and the constants from the homogenous solution can be determined using the displacement and slope angles at both ends of the beam. When the beam remains linear in terms of the stress-strain relationship, conditions of slope can be replaced with conditions of strain (at the upper/lower surface):

$$u(x_C) = f(u_A, \varepsilon_A, u_B, \varepsilon_B) \qquad (2)$$

Where f denotes a certain function. This expression is useful in cases that the strain instead of slope is measured. When $EI(x)$satisfies certain conditions such that $u(x)$ is linear in its homogeneous constants, f becomes a linear function.

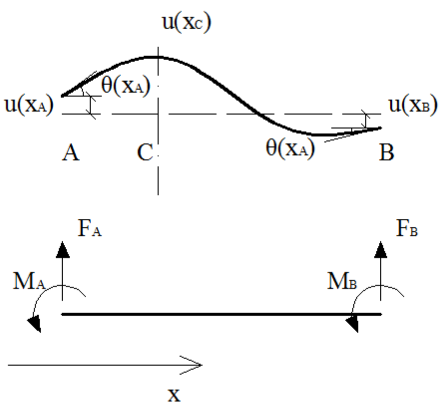

Figure 1 The deflected shape and free body diagram of the static beam element model

In dynamic vibration monitoring settings that measure the systems acceleration instead of displacements, this model can still be applied by taking the 2nd derivative of Eq. (2) if the system stiffness-to-mass ratio is large:

$$a_C = \ddot{u}(x_C) = f(\ddot{u}_A, \ddot{\varepsilon}_A, \ddot{u}_B, \ddot{\varepsilon}_B) = f(a_A, \ddot{\varepsilon}_A, a_B, \ddot{\varepsilon}_B) \qquad (3)$$

This relation can also be formulated in the frequency domain by taking the one-sided Fourier Transform:

$$\hat{a}_C(i\omega) = \hat{f}(a_A, \ddot{\varepsilon}_A, a_B, \ddot{\varepsilon}_B) \xrightarrow{f\ linear} f\left(\hat{a}_A(i\omega), \hat{\ddot{\varepsilon}}_A(i\omega), \hat{a}_B(i\omega), \hat{\ddot{\varepsilon}}_B(i\omega)\right) = f\left(\hat{a}_A(i\omega), -\omega^2\hat{\varepsilon}_A(i\omega) - i\omega\varepsilon_A(t=0) - \dot{\varepsilon}_A(t=0), \hat{a}_B(i\omega), -\omega^2\hat{\varepsilon}_B(i\omega) - i\omega\varepsilon_B(t=0) - \dot{\varepsilon}_B(t=0)\right) \qquad (4)$$

Beam model with a lumped mass

The model introduced in the previous subsection addresses only static/quasi-static applications. Here a model that incorporates a part of dynamic effects is constructed by adding a lumped mass in to the beam(Fig. 2). Again assuming that the material constitutive relation is linear and applying the generalized force concept:

$$u(x) = f(u_A, \varepsilon_A, m\ddot{u}_C, u_B, \varepsilon_B) \qquad (5)$$

when f is a linear function (condition for this to hold is still the same as in part (1)), Eq. (5) can be reformulated as

$$a_C = \ddot{u}_C = f_2(u_A, \varepsilon_A, u_C, u_B, \varepsilon_B) \qquad (6)$$

f_2 is another function with different coefficients. Its corresponding frequency domain representation is

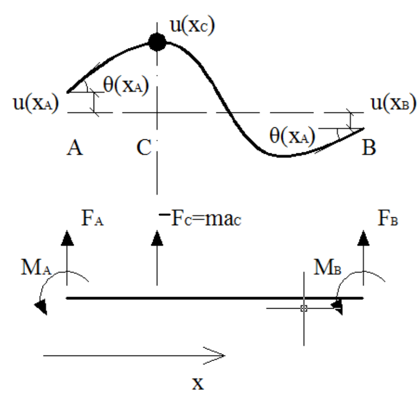

Figure 2 the deflected shape and free body diagram of the lumped mass model

$$\hat{a}_C(i\omega) = \hat{f}_2(u_A, \varepsilon_A, u_C, u_B, \varepsilon_B) \xRightarrow{f\ linear} f_2\big(\hat{u}_A(i\omega), \hat{\varepsilon}_A(i\omega), \hat{u}_C(i\omega), \hat{u}_B(i\omega), \hat{\varepsilon}_B(i\omega)\big)$$
$$= f_2\left(\frac{\hat{a}_A(i\omega) + i\omega u_A(t=0) + \dot{u}_A(t=0)}{-\omega^2}, \hat{\varepsilon}_A(i\omega), \frac{\hat{a}_C(i\omega) + i\omega u_C(t=0) + \dot{u}_C(t=0)}{-\omega^2}, \frac{\hat{a}_B(i\omega) + i\omega u_B(t=0) + \dot{u}_B(t=0)}{-\omega^2}, \hat{\varepsilon}_B(i\omega)\right) \quad (7)$$

Note that when the substructure is subjected to the ambient load/white noise load, the two models described can still be applied by using the correlation of the signals with the regressand signal as a free-decay response. The corresponding frequency domain relation will then be defined for the auto/cross power spectral densities, instead of the Fourier transform.

FORMULATION OF DAMAGE DETECTION ALGORITHM FROM LINEAR REGRESSION TECHNIQUES

The general MISO(multi-input-single-output) linear regression problem can be formulated as below:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

Where **Y** is the $n \times 1$ regressand vector, **X** is the $n \times m$ regressor matrix, β is the $m \times 1$ coefficient vector, and ϵ is the $n \times 1$ residual series. Here in the size definitions *n* stands for the number of observations, and *m* refers to the number of input series. Thus given the input and output observations, the coefficients and the residuals can be estimated through applying standard curve fitting algorithms with a certain criteria (often least squares).

Henceforth, two categories of linear-regression-based damage detection algorithms can be formed: coefficients-based method and residual-based method. The former requires repetition of the coefficient estimation process in order to collect enough samples for coefficients values postprocessing, the latter, on the other hand, can function with only one set of estimated coefficients from the baseline/damaged state, and residuals of all the data sets to be processed will be obtained using this set of coefficients. Statistical moments of the residuals are commonly used as viable damage indices.

The substructural models presented in the previous section can thus be used to form regression models in Table 1. Note that for all the regression schemes only acceleration and strain signals are employed as they are most commonly measured vibrational responses.In the table, subscript *j* is a time label range from 1 to *N*,which is the total number of sample points in time/frequency domain depending on the situation. $\Delta^2\{\cdot\}$ represents the central difference of a signal, $\iint\{\cdot\}$ denotes the reconstructed displacement from the acceleration signal inside the brackets by applying the CFIR filter described in [7], and the macro accent $\overline{\{\cdot\}}$ means the detrended signal. The purpose for detrending is to eliminate from the regression models the regression constant, which is associated with the system static deformations in the time domain signal,and the vibration intial conditions in the one-sided frequency spectrum.

Table 1. the linear regression models derived from the substructural beam models

Model type	Choice of regressors/regressand
1. Static beam model (Time domain)	$\mathbf{Y} = \mathbf{a}_C(t_j), \quad \mathbf{X} = [\mathbf{a}_A(t_j), \Delta^2\{\bar{\boldsymbol{\varepsilon}}_A(t_j)\}, \mathbf{a}_B(t_j), \Delta^2\{\bar{\boldsymbol{\varepsilon}}_B(t_j)\}]$,
2. Static beam model (Frequency domain)	$\mathbf{Y} = Re\{\bar{\hat{\mathbf{a}}}_C(i\omega_j)\}, \mathbf{X} = Re\{[\bar{\hat{\mathbf{a}}}_A(i\omega_j), -\omega_j^2\bar{\hat{\boldsymbol{\varepsilon}}}_A(i\omega_j), \bar{\hat{\mathbf{a}}}_B(i\omega_j), -\omega_j^2\bar{\hat{\boldsymbol{\varepsilon}}}_B(i\omega_j)]\}$,
3. Beam model with lumped mass (Time domain)	$\mathbf{Y} = \mathbf{a}_C(t_j), \mathbf{X} = [\iint \mathbf{a}_A(t_j), \bar{\boldsymbol{\varepsilon}}_A(t_j), \iint \mathbf{a}_C(t_j), \iint \mathbf{a}_B(t_j), \bar{\boldsymbol{\varepsilon}}_B(t_j)]$,
4. Beam model with lumped mass (Frequency domain)	$\mathbf{Y} = Re\{\bar{\hat{\mathbf{a}}}_C(i\omega_j)\}, \mathbf{X} = Re\left\{\left[\frac{\bar{\hat{\mathbf{a}}}_A(i\omega_j)}{-\omega_j^2}, \bar{\hat{\boldsymbol{\varepsilon}}}_A(i\omega_j), \frac{\bar{\hat{\mathbf{a}}}_C(i\omega_j)}{-\omega_j^2}, \frac{\bar{\hat{\mathbf{a}}}_B(i\omega_j)}{-\omega_j^2}, \bar{\hat{\boldsymbol{\varepsilon}}}_B(i\omega_j)\right]\right\}$.

In this paper, the estimated regression coefficients ($\hat{\boldsymbol{\beta}} = (\mathbf{X}\mathbf{X}^{\mathrm{T}})^{-1}\mathbf{X}^{\mathrm{T}}\mathbf{Y}$ from the least squares method), the ratio of the variance of regression residuals (from baseline model) to that of the signal ($RF1 = var(\boldsymbol{\epsilon}_{BL})/var(\mathbf{Y}), \epsilon_{BL} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{BL}$) and the ratio of the residual variance from fitting the baseline coefficients to that from fitting the estimated coefficents from the current state ($RF2 = var(\boldsymbol{\epsilon}_{BL})/var(\boldsymbol{\epsilon}_{CS}), \epsilon_{BL} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{BL}, \epsilon_{CS} = \mathbf{Y} - \mathbf{X}\hat{\boldsymbol{\beta}}_{CS}$).

CHANGE POINT ANALYSIS TECHNIQUES

To effectively identify damage in the structure, proper damage threshold construction methods should be devised for the features extracted. Here change point analysis is adopted to determine the threshold of all features presented above. Namely, change point analysis (CPA)[8] refers to a family of methods that detects in a series of values the point where the statistical properties of the data change. In many practices of quality control, it is often used in conjunction with statistical control charts; the former is data intensive approach and thus more robust to noise, the latter is an instantaneous method for damage identification and more sensitive to short-period variations. Here as all the features are based only on structural output (i.e. acceleration signals), change point analysis is the preferred choice for stability reasons.

In the following subsections, two methods that identifies shift in the mean of the signal will be described. The first method finds the change point as where the maximum absolute value of the signal cumulative sum occurs, and the second one locates the point as that minimizes the deviation of the entire signal.

The cumulative sum based CPA

Using this method, the change point of a data sequence $\{x_i\}_1^L$ can be computed in 3 steps:

(1) Subtract the sequence by its mean. ($\{x_i'\}_1^L = \{x_i\}_1^L - \bar{x_i}$)
(2) Calculate the cumulative sum at each data point. ($S_i = \sum_1^i x_i{'}, 1 \leq \ i \leq L$)
(3) Identifies the change point as the where the maximum absolute value of cumulative sum is found. ($CP = \mathrm{argmax}_i(|S_i|)$)

A bootstrapping technique is used to determine if the change is statistically significant at level α:

(1) Randomly permute the original sequence. ($\{\tilde{x}_i\}_1^L = randperm(\{x_i\}_1^L)$)
(2) Compute and record the maximum absolute value of cumulative sum for this new sequence (with all values subtracted by the mean).
(3) Repeat Step 1-2 for N times, and use the value that's beyond $N(1-\alpha)$ values in the pool as the threshold.

If $\max(|S_i|)$ of the original sequence is larger than the threshold. The change point is deemed to be statistically significant.

The deviance reduction based CPA

The deviance reduction based method identifies the change point as the place where largest deviance variance reduction is obtained by splitting the data into two parts:

(1) Compute the deviance for the entire signal. ($Dev_t = \sum_1^L \left(x_j - \bar{x}|_1^N\right)^2$)
(2) At each data point, split the sequence into two parts and compute the resulted deviance reduction. ($DR = Dev_t - Dev_{1:i} - Dev_{i+1:L} = \sum_1^L \left(x_j - \bar{x}|_1^N\right)^2 - \sum_1^i \left(x_j - \bar{x}|_1^i\right)^2 - \sum_{i+1}^L \left(x_j - \bar{x}|_{i+1}^N\right)^2$)
(3) The change point is identified as the split point that yields the maximum deviance reduction. ($CP = \mathrm{argmax}_i(DR)$)

Again, bootstrapping is used to determine the significance threshold. The general methodology is similar to that in the cumulative sum method, only that here the values computed and recorded are deviance reductions of permuted data sequences, rather than the maximum absolute values of the cumulative sum.

In all the applications presented herein, only one change point needs to be found as only one change of structural state occurs for each case study. When there is more than one possible change points in the data sequence, these methods can be used in a recursive manner.

Two indicators here are adopted to decide on the statistical significance of the change; the normalized damage indication variable(NDIV) with normalizing window shown in Fig. 3, and the ratio of the mean shift of the two sample groups separated by the change point selected to the average of the variances of the two groups (normalized mean shift, NMS).

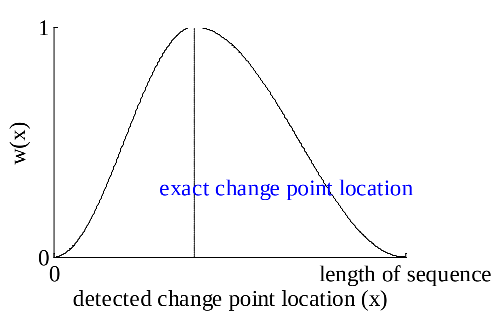

Figure 3 Normalizing window for NDIV; both sides of it are of raised cosine shape

NUMERIAL VERIFICATION OF THE PROPOSED ALGORITHMS

A small-scale two-span steel bridge girder simulated in SAP2000 is used here for verification of the proposed damage detection methodology. The girder was modeled as a two-dimensional frame with coil-spring-constrained supports and uneven spans as shown in Fig. 4. The uneven spans allow for more variety in the results and damage scenarios. The coil spring constants were chosen so that the behavior of the frame will resemble that of the real girder built in the testing lab. The reason to choose the simulated model here is to obtain data at various sensing locations, as not so many strain gages are available on the real specimen. The model has 23 total nodes, which coincide with the accelerometer/strain gage locations.

For vibration data collection, a white noise excitation was applied in the horizontal direction to produce responses at each node. Measurement noise was accounted for by adding 5% random noise to the response. Four damage scenarios are simulated in succession by switching out an 20.32 cm portion at distinct sensor location 6, 15, 17 and 20, respectively. Except that the switched out portion is replaced with another tube with only 50% of the original section stiffness, the rest of the structure maintains the same stiffness properties as the undamaged. For testing of each structural state (undamaged/damaged), acceleration and strain signals are simulated at 500 Hz sampling frequency.

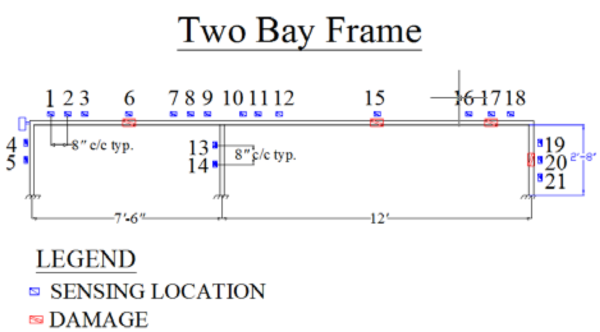

Figure 4 the sensing scheme for the two-bay frame model

Damage identification--change point histograms

For each damage scenario, the algorithms outlined in Section 2-4 are applied. When applying frequency domain techniques, only samples larger than the median response are used for noise-robust performance.The combination of 4 regression models, 3 damage indices and 2 CPA techniques yield 4×3×2=24 ways to identify damage. For each type of damage index, the values extracted from 10 sets of signals collected from damaged state are compared with those from 10 sets of baseline signals. Thus the ideal case is that all damage indices that reports damage though CPA shows a change point at 10. However, in the results acquired from applying the damage localization algorithm to the data, the histogram of change point locations have a wide spread, and the correct change point location needs be recognized through a statistical inference , i.e. taking the first moment. The errors are probably caused by noise from sensing measurements, the effect of which tend to get larger for complex models with a number of paramenters. Another way to counter the interference of large noise variance is to collect more vibration signals so as to have more estimated damage feature samples for the CPA.

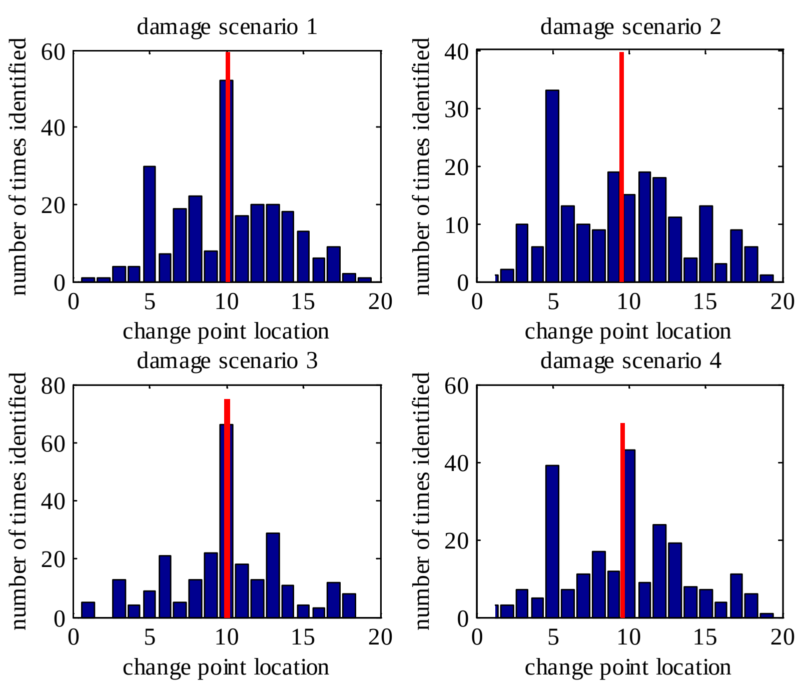

Figure 5 CPA results for the four damage scenarios; red bar shows the weight mean location

Damage localization – identification of the location where the most significant change occurs

Table 1 summarizes the damage localization results for all 4 different damage states. Basically, the sensor (or sensor pair) location that corresponds to the largest change in damage location indicator values are identified as the damage location. When multiple sensor/sensor pair location tied for the largest damage location indicator value, they are all retained as possible damage location if the assocaiated features are residual-based, but for regression coefficient features multiple sensor pairs are only allowed if the regression coefficients are from the same substructural model. Otherwise, tie-breaking rule that checks changes for other coefficients in respective models are employed. During the data processing step, the indicator values of damage indices that are from the same regression model type and category (i.e. either coefficients/residual based)but different extraction functions (if there are) and CPA methods are summed together as they tend to be highly correlated. This would also make comparison and contrast among the results an easier task.

Table 2. Damage locations as determined from multiple algorithms

	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
	Damage scenario 1 (at location 6)				Damage scenario 2 (at location 15)			
NDIV from regression coefficients*	6-7	6-7	3-6-7	3-6-7	3-2	7-8	12-15	17-16
NMS from regression coefficients*	3-6	6-7	3-6-7	3-6-7	3-2	7-8	12-15	5-5
NDIV from RF1and RF2	7	6	6	6/11	16	12	12	15
NMS from RF1and RF2	7	6	6	6	16	12	12/15	17
	Damage scenario 3 (at location 17)				Damage scenario 4 (at location 20)			
NDIV from regression coefficients*	16-17-18	16-17	6-7	12-15	21-20	21-20	14-13	10-11-12
NMS from regression coefficients*	16-17	16-17	16-17	15-16	21-20	21-20	12-11	11-10
NDIV from RF1and RF2	16/17	16/17	21	15/16	21	2	21	21
NMS from RF1and RF2	16	16	21	15	21	21/12	21	21

* when two numbers are hyphenated, the first number denotes the regressand node; when three numbers are hyphenated, the middle number denotes the regressand node.

None of the methods proposed has a 100% correct performance as the simulated structure iis different from the models presented. It can be seen that overall the residual-based methods perform better than the coefficient based damge methods in damage localization, and that the dynamic models outperform the static models for longer/suppler beam substructural models because the inertia force is accounted for, but underperform for shorter/stiffer beam models because their dynamic behavior is not significant. Therefore it is concluded that there is a trade-off between model accuracy and over-parameterization when designing damage detection algorithms. Also, it is noticed that the results would also improve for damaged locations closer to the excitation source (such as location 6) for the larger signal-to-noise ratios.

A direct voting scheme among the results from different type of methods[9] is used to decide on the most probable location of damage; 1) assign 1 weight to the locations indicated by residual based methods and the regressand nodes of coefficient-based methods, and 0.5 weight to indicated by the regressor nodes of coefficient-based methods. 2) pool the votes together to select three locations with the top scores.Number 3 is used as theoretically this is the maximum number of substructural models that will be affected by a potential damage. The results are shown in Fig. 6.

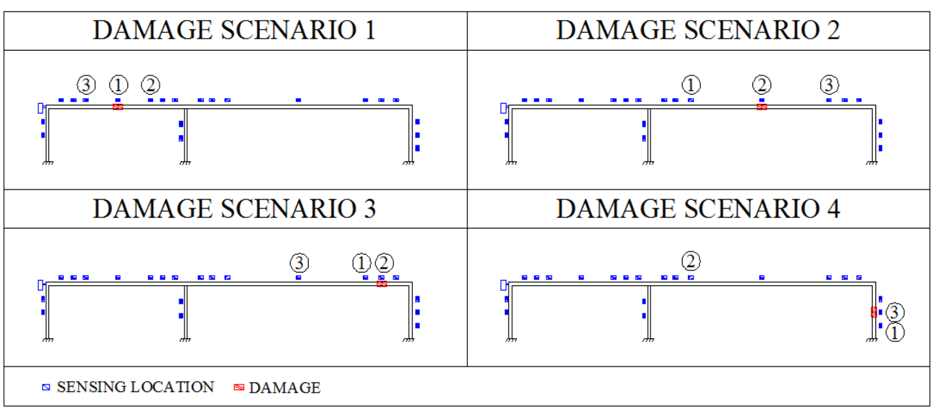

Figure 6 Inferred damage location from combining classifiers

CONCLUSION

In this paper, several substructural-based linear regression schemes are presented and adopted for damage identification and localization in a simulated steel frame specimen. Four substructural models are proposed, and in regard to each model three damage features, one as regression coefficients, two as functions of regression residuals, can be extracted. Two change point analysis (CPA) techniques are then used separately to determine if a statistically significant damage occurred in sequences of certain extracted damage features. All twenty-four classifiers thus obtained through joining individual model construction, feature extraction and CPA algorithms are applied to data from the simulated frame, which is set to undergo four different damage scenarios.

Application results show that the suggested algorithms succeeded in identifying the damage existence for all four damage states. However, the exact location of the change point, which indicates when the structure is damaged, is not obvious from viewing the results and has to be determined through calculating the weighted mean. This is because of the models adopted here have higher complexity than those in the previous research projects[10-12] and aim to extract information pertaining to higher level damage detection (damage localization). As such, the information-to-noise level here is reduced as a result of the combined noise effect from several signal sources and the reduced signal strength from multi-level data preprocessing. In light of this phenomenon, care must be taken for model selection to avoid overparameterizaton.

For damage localization, the substructural approaches are shown to be effective. The procedure much resembles an inversed finite element analysis approach, where the structural properties are estimated from structural responses through assuming that the structure can be represented mathematically by an assemblage of elements. it is observed that the residual-based methods are more sensitive than their coefficients-based conterparts. This observation is in line with the previous work, and the fact that the change in residuals properties is related to changes in all regression coefficients. On the other hand, when the excitation conditions are varying, the residual-related features will have a larger chance to be affected. Also noticed is that the algorithm performance is enhanced when the signal strength at the damage location is large.

It is found that more accurate decisions on damage location can be made when results from several methodologies are combined through a weighted voting scheme. This is expected as the errors from simplified linear modeling are migitated through the 'averaging process', and combined classifiers have proved a success in many other pattern recognition fields[9]. Still, further research are needed to determine most effective and efficient ways for the combination.

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