Response Surface Model Updating for Nonlinear Structures

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ABSTRACT- This paper presents a procedure to update nonlinear finite element models in time. In the proposed method, accurate response surface models are constructed and evaluated to replace the finite element model at every time step of the analysis. Then, the optimization problem of model updating is formulated and solved iteratively leading to histograms of the updated model parameters. This methodology is beneficial in extracting more information from measured signals and compensate for the error present in the regressed response surface models. The proposed method was verified through a numerical case study of a steel frame with global nonlinearity. Appropriate design and model orders were successfully established and the optimization in time performed well in the simulated scenarios under the assumption of noise free and noisy measurement data.

Keywords: Finite element model; Model updating; Nonlinear structure; Response surface model; Optimization

INTRODUCTION

Finite element (FE) models are extensively used for analytical purposes in the engineering field. These models serve as the preliminary base for designing and analysing behaviour of the actual structures. However, the analysis results of these models are not same as that obtained from testing the actual structures. This is mainly because of a number of simplifying assumptions used in the FE model or/and deterioration of the structure with use and time. Modifying the FE model to obtain better agreement with the experimental records is termed as *Finite Element Model Updating.* Over the past decades several computational procedures have been developed to update parameters of analytical models based on experimental results. Methods for linear model updating are well-documented in the literature. These techniques are mainly based on the sensitivity analysis and linearization of the generally nonlinear relationship between measured outputs such as natural frequencies, mode shapes or displacement responses and the parameters of the model in need of correction [1]. However, iterative determination of local gradient in such methods may cause not only computational intensive, but also convergence difficulty [2]. Moreover, in the presence of nonlinearities in the structure these procedures will fail to yield the parameters associated with nonlinear behavior of the model and other measures are required to update the model.

One of the proposed approaches to overcome these problems is to replace the FE model with a mathematical expression which approximates the relationship between pre-selected inputs and output of the FE model and update the parameters of the model by directly optimizing this surrogate model. One of the commonly used surrogate models are polynomial functions constructed based on Response Surface (RS) methodology. This method has shown attractive potential in modifying FE model parameters. Guo and Zhang [3] found that, compared with the sensitivity-based model updating, the RS-based method gave likewise accurate predictions while requiring much fewer number of FE analyses. Ren and Chen [2] compared the performance of RS-based and Sensitivity-based FE model updating on a full size precast continues box girder bridge and observed that the rate of convergence in RS-based updating is faster. Ren et al [4] concluded that for complex structures with large number of uncertain parameters uniform design economized the computational effort to construct the RS models and the accuracy of the RS models in such problems could be improved by shrinking the design space and repeating the RS modeling and updating. Zhang et al [5] proposed a model updating technique based on generic algorithm and RS methodology. Application of the method on a numerical simulation of an antenna successfully reached the global optima.

Studies of Cundy [6] and Fang and Perera [7] found that application of RS-based model updating in damage detection performs well in locating damage and quantifying its severity to some extent in numerical and experimental case studies. There are few examples of application of RS-based model updating in the literature for structures with nonlinearities. Schultze et al [8] applied this method to select significant parameters to update a model consist of a cylindrical steel impactor and a foam layer assembled on a mounting plate attached to a drop table under impact on a concrete floor. Zhang and Guo [9] proposed a model updating procedure based on Principal Component Decomposition and RS method to update a model of frame with thin wall components showing strain-rate-dependence nonlinearity under impact test.

In this paper a procedure is proposed to update nonlinear FE models in time. For this purpose, low computational effort associated with RS modeling is used to formulate and solve the optimization problem of model modification in the length of time domain data iteratively. This approach is beneficial in extracting more information from the measured experimental signals as opposed to the traditional approaches in which the whole measured signals are summarized into one or more response features. Another advantage of this method is that it is not limited to the type of model behavior or analysis. It can be applied to linear or nonlinear models under static or dynamic analysis. Since the procedure of finding an appropriate design to build accurate RS models requires a number of trials and errors with different designs and subset models, a procedure is also proposed to design the levels of input parameters and construct the RS models prior to model updating. This procedure results in RS models capable of generating the results of FE analysis in a specific domain of input variables.

In the following sections of the paper the proposed procedure is explained and the results of application of this method on a numerical case study are demonstrated.

NONLINEAR MODEL UPDATING USING RESPONSE SURFACE MODELS

To update nonlinear FE models through time history of measured responses, in every time step of the analysis a RS model is constructed to produce the response of the FE model at that time step. To do so, the experimental input force is used to generate the equivalent responses of FE model at different levels of the model parameters. These levels of the model parameters, corresponding responses of the FE model and least square estimation techniques are used to find the best polynomial model which can replace the FE model at every time step of the analysis.

Eq. (1) denotes the RS model at the l^{th} time step of the analysis, where h is the polynomial surrogate model in that time step and Θ represents a vector of model parameters selected for modification.

$$RS_l = h_l(\Theta) \tag{1}$$

By completing this process for every response, an objective function is formulated to minimize a function of residuals of RS-based and experimental response features at every time step. Eq. (2) represents this minimization problem which is solved inside the domain of model parameters.

$$\min_{\theta} \ f_l(H_l(\Theta), Y_{\exp l}) \tag{2}$$

In Eq. (2) H_l and $Y_{\exp l}$ are vectors containing all the surrogate models and corresponding experimental responses at the l^{th} time step.

Prior to RS modeling, the appropriate design and model order should be found so that the regressed RS models are accurate at the associated time steps. First, an initial region for the pre-selected uncertain parameters of the FE model should be chosen. This region, in which the FE model is replaced by the RS model, is called *RS domain*. To regress the polynomial RS models, a number of points are sampled in the RS domain based on full factorial design of the model parameters. The RS model construction starts with a full factorial design with three levels for each parameter and including linear terms of the updating parameters in the RS models. Initially the performance of the RS models is checked at the design points based on the residuals of the RS and FE models. Then the overall adequacy of the RS models is evaluated by adjusted R^2 statistics. If R^2_{adj} is close to one, it implies a perfect regression. Therefore, when R^2_{adj} is much smaller than one, the RS model is not accurate in estimating the FE responses at the design points. After completing R^2_{adj} calculation through the time domain data, if the regressed RS models are not fitted well to the design points, higher order terms of the model parameters should be added to the RS models and the model evaluation repeated to find the appropriate model order.

After finding the suitable model order, the prediction quality of the RS models should be checked. For this purpose, residuals are calculated at points in the RS domain that did not contribute in the regression. These points, which are called intermediate points, are sampled from RS domain in different sets. Each set represents the intermediate levels for one parameter. To sample a set of new points corresponding to a parameter, one of the original data points is replicated, and then the selected parameter is replaced by the average of one pair of its original levels. Intermediate points which result in larger residuals than the original design points indicate that although the RS model has been fitted well to the original data, it cannot predict the FE responses for new points. Therefore, the design of levels of parameters should become finer and new RS models should be regressed.

By repeating this procedure, an RS model with high quality in regression and prediction is constructed for every time step of the data. Upon completion of this procedure for every response feature, the optimization problem of model updating is solved for every time step which results in histograms of the updated model parameters. The optimization step can be repeated in a smaller region for model parameters centred on the mean value of the results of the first cycle of optimization. Using the design and model order established in the first cycle, only the following steps are needed: (1) generate the FE responses for new levels; (2) fit the new RS models through the time history; and (3) optimize the new objective function iteratively.

CASE STUDY: NONLINEAR STEEL FRAME

The case study presented here is a steel frame with nonlinear material properties under dynamic loading. The frame consists of one span with overall length of 228.6 cm supported by columns that are 83.8 cm long. The cross section of the beam and column members is uniform hollow 5.08 cm tube, with 0.21 cm wall thickness. The steel has bilinear behavior with the yield stress of 344.8 MPa. Modulus of elasticity (E) and post yielding stiffness ratio of steel (b) were chosen as the updating parameters. To simulate the experimental data, these parameters were set to 193.1 GPa and 0.18 for E and b respectively. The loading is a concentrated harmonic lateral load with amplitude 22.2 kN and 5 sec period, applied at the beam column joint. The amplitude of the load is selected so that under lateral loading the stress in the columns and beam exceeds the yield stress. To update the selected parameters, simulated time histories of displacement at two locations on the frame were assumed as the experimental data. Fig. 1 shows the configuration of the steel frame, loading and the responses used in the updating procedure.

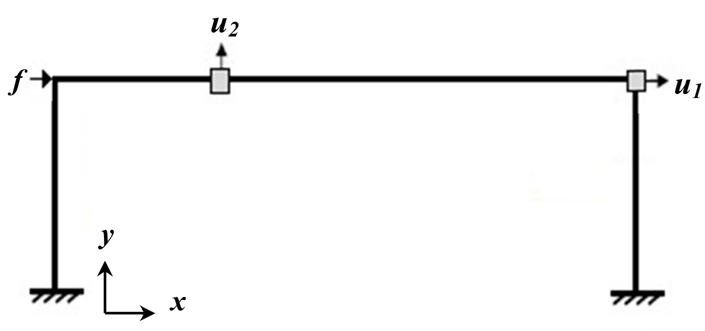

Fig.1: Configuration of the nonlinear steel frame

A 2-D FE model was developed by Opensees software using fiber section procedure and Steel01 uniaxialMaterial properties. The initial domain of the updating parameters was set to 186.2 to 227.5 GPa for E and 0.05 to 0.25 for b. The RS model construction starts with full factorial design of parameters, each having three levels. RS models including the linear terms of E and b were regressed to the data at every time step. The large residuals associated with the regressed models indicate that the RS models are not accurate to replace the FE model. Consequently, quadratic terms were added to the polynomial models and regression was repeated. Adding the quadratic terms to the linear models significantly improves the accuracy of the RS models at the design points. Fig.2 and 3 compare the maximum normalized residuals at original and intermediate points of the 3×3 design for the quadratic RS models through the time data. The RS models generate u_2 with smaller residuals; however, they are not successful in predicting both u_1 and u_2 at the intermediate levels corresponding to b. Therefore, the levels associated with stiffness ratio, b, in the RS domain should be finer.

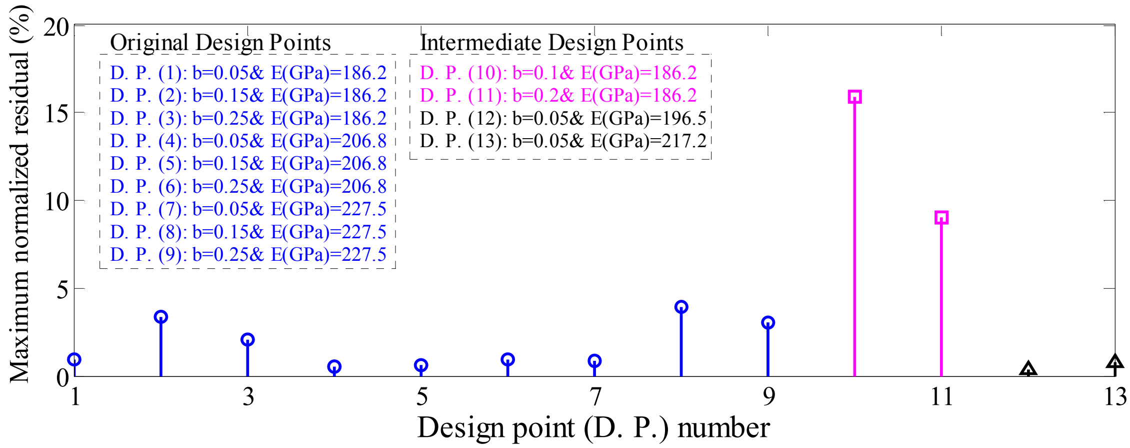

Fig.2: Normalized residuals of original and intermediate design points using 3×3 design: u_1

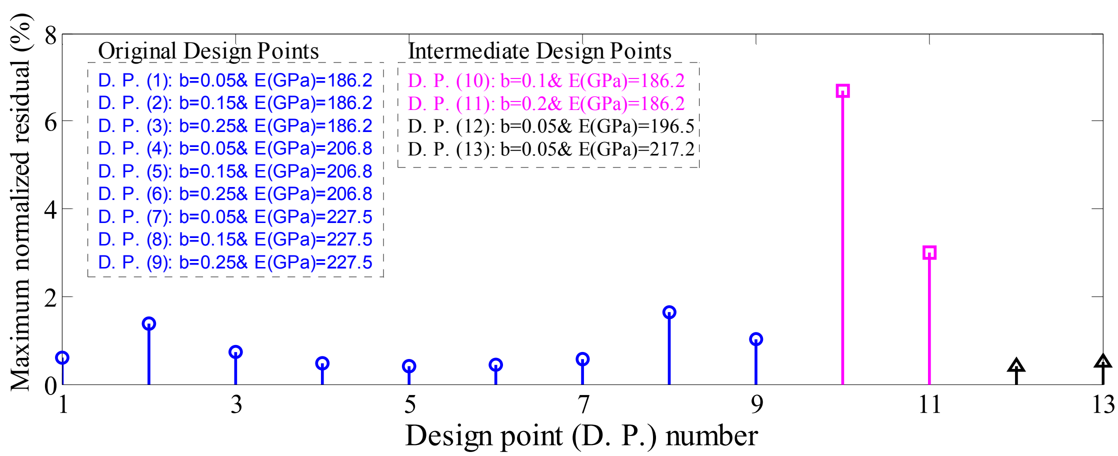

Fig.3: Normalized residuals of original and intermediate design points using 3×3 design: u_2

RS model construction and evaluation were repeated with 4×3 design and observed that the RS models are not accurate at intermediate levels of parameter b for this design. Fig.4 and 5 displays the maximum normalized residuals of RS and FE model for a 5×3 design. The RS models contain terms up to order 4 and 2 for stiffness ratio, *b*, and modulus of elasticity, *E*, respectively. These figures show that the RS models perform well at both original and intermediate levels. Therefore, the RS models are accurate for the optimization procedure.

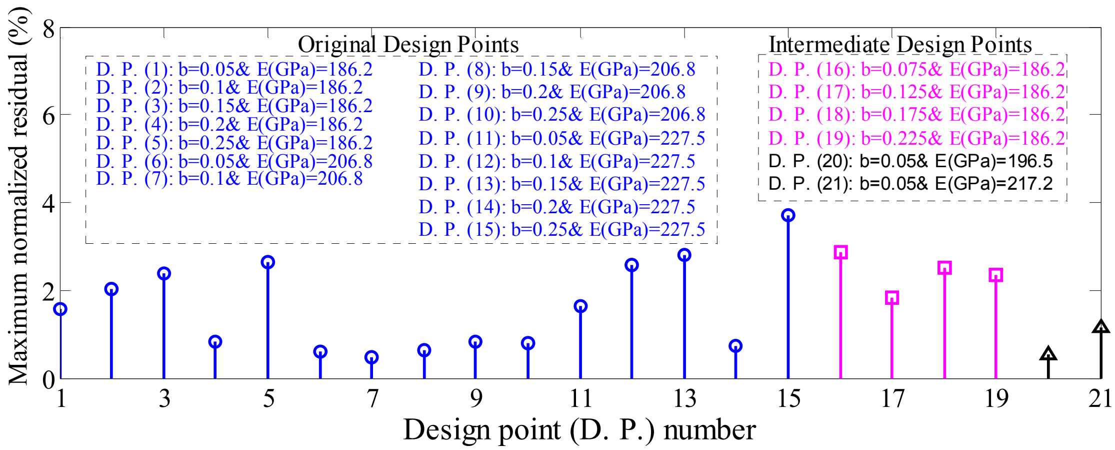

Fig.4: Normalized residuals of original and intermediate design points using 5×3 design: u_1

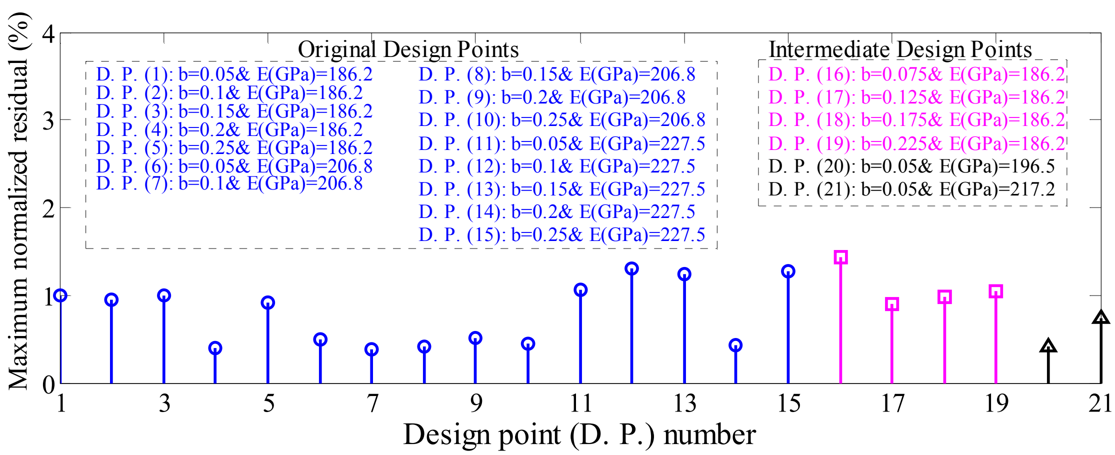

Fig.5: Normalized residuals of original and intermediate design points using 5×3 design: u_2

Fig. 6(a) shows the histogram of the updated parameters resulted from solving the optimization problem in every time step for the first time. This histogram shows where the updated parameters locate in the RS domain. The updated model parameters are distributed in a considerably narrower region than the initial region used in the RS model construction. To decrease the variation of the updated parameters, the design and model order established in the previous section for E and b are used to repeat the optimization problem. The new domain for E and b is centered on the mean value of the updated parameters in the first round of optimization. Fig. 6(b) shows the result of the second round of model updating in terms of the distribution, mean and coefficient of variation of the updated parameters.

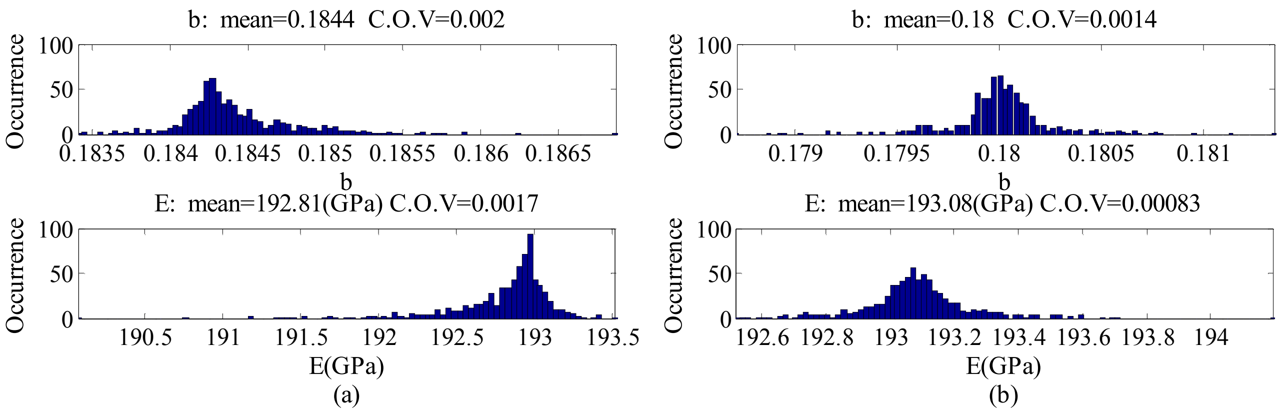

Fig.6: (a) Results of the first optimization round, (b) Results of the second optimization round

Furthermore, to evaluate the performance of the proposed procedure in the presence of noisy measurement data, different levels of Gaussian noise were introduced into the simulated experimental responses and the updating procedures were reiterated. The results were summarized in Table 1. Noise level in this table represents the ratio of the root mean square of the simulated noise signal to the root mean square of the original signal. It can be seen that the procedure shows robustness to different levels of noise.

Table.1: Comparison of the performance of the proposed procedure (Simulated data with different noise levels)

	True model parameters: b=0.18 & E(GPa)=193.1		
		Updated Parameters	
	Noise Level (%)	b	E
		Relative Error (%)	
b: 0.05 to 0.25 E(GPa):186.2 to 227.54	0	0.00	0.00
	1	0.11	0.11
	5	-1.50	1.57
	10	-3.06	3.07

The advantage of using the proposed procedure for updating nonlinear FE models is that this method successfully finds a smaller region for the model parameters and has corrective information for the initial estimate of the RS domain. Moreover, it is computationally efficient and shows robustness to moderate and high level noise.

CONCLUSIONS

This paper presents a procedure for updating nonlinear Finite Element models using time domain data based on Response Surface methodology. The proposed procedure was validated through to a numerical case study of a steel frame with global nonlinearity. The optimization in time domain performed well in the numerical case study. The first round of optimization resulted in a considerably narrower bounds for the uncertain parameters of the model than the initial boundaries set at the beginning of the procedure. Repeating the RS model construction with known order and design for the new bounds of parameters and solving the optimization problem resulted in the true model parameters. To verify the robustness of the results, the numerical case study were repeated assuming low, moderate and high level noise in the experimental data and it was observed that the proposed procedure show robustness in all these scenarios.

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