COVER SHEET

Title: *Performance comparison of different autoregressive damage features using acceleration measurements from a truss bridge*

Authors: Ruigen Yao
Shamim N. Pakzad

ABSTRACT

Time series analysis has been applied to structural monitoring signals for system damage identification in a number of research literatures. Among various time series analysis tools, univariate autoregressive modeling (AR) is one of the most commonly used methods because of its innate computational efficiency. In this paper, three autoregressive damage features extracted directly from the ambient vibration data and from the vibration signal autocorrelation will be presented. Two of the features are distance functions of AR model parameters and the third feature is a function of AR residuals. These features are then applied to acceleration measurements collected from a member of a truss bridge to detect a structural change, and their performances are compared and commented.

INTRODUCTION

Damage detection is a very crucial part in the regular assessment and maintenance routine for civil infrastructure. Traditionally this task is carried out by human inspection, and thereby is expensive, time consuming, and most importantly, the accuracy relies on individual expertise. Recently, the advancements in sensing and computational technology have given rise the hope that a sensor network can be installed on a civil structure, and data collected from the sensors will be processed for information pertaining to the structural condition. To date many research reports and papers [1-3] devoted to this topic can be found, forming a promising branch of study often referred to as data-driven structural health monitoring (SHM). Ideally, the new system will cost less than traditional method because of the reducing prices of electronics products, and produces more reliable decision that is free of human judgment bias.

Ruigen Yao, Graduate Student, Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, PA 18015.
Shamim N. Pakzad, Assistant Professor, Department of Civil and Environmental Engineering, Lehigh University, Bethlehem, PA 18015.

Vibration responses (e.g. acceleration, strain) are among the most commonly measured signals for structural monitoring purposes. Many damage features based on structural vibration responses are proposed to extract useful structural information and identify possible damage. Time series analysis for single channel acceleration measurements is one of the notable methods attempted in a number of research articles [4-7], where algorithms such as scalar autoregressive (AR), autoregressive/autoregressive with exogenous input (AR-ARX), autoregressive with moving average (ARMA) modeling have been applied and functions of estimated model parameters used as damage features. These features are reported to be less complicated to compute and more sensitive to local damage.

Though there is an abundance of features proposed on structural damage detection, cross-comparison of their performances on different types of structures are needed to determine the merits and deficiencies of each damage feature. In this paper, the Mahalanobis distance of AR coefficients, Cosh distance of AR model spectra and Q-statistic of AR residuals will be applied to detect a structural change for one of the structural members of a bridge over the Allegheny River in Pennsylvania, which was then under retrofit. These three different damage features can be estimated directly from ambient vibration signals, and in this paper it is proposed that they also can be estimated from the signal auto-correlation functions (ACF). Results from the three features estimated using both strategies are presented and compared.

The paper is organized as follows: Section 2 gives an explanation of the univariate AR models for random signals (e.g. ambient vibration) and deterministic processes (e.g. signal autocorrelation function), together with the extraction procedure of the three damage features. Section 3 contains descriptions of the bridge, the data collection scheme, and damage detection results using these features. Conclusions are then made on the respective effectiveness of the features.

AUTOREGRESSIVE MODELING FOR STRUCTURAL DAMAGE IDENTIFICATION

The univariate AR model is a widely used prediction model with several efficient algorithms available for the model parameter estimation. AR model for stationary random signals [8] can be expressed as:

$$x(t) = \sum_{j=1}^{p} \varphi_{xj} x(t-j) + \epsilon_x(t). \qquad (1)$$

In this equation, $x(t)$ is the time series under consideration, φ_{xj} are the model coefficients, and $\epsilon_x(t)$ is the model residual. This model predicts the current value of a signal as a weighted combination of its previous values up to model order p. In the subsequent application, the Yule-Walker method will be adopted to estimate the AR coefficients directly from stationary series.

Deterministic stable processes with a state-space representation are perfectly predictable from a finite number of past values [9]. Here for structural vibration measurements from a single sensing location, their ACF, $R(t)$ is essentially a sum of M exponentially decaying sinusoids [10],

$$R(t)=\sum_{m=1}^{M} A_m e^{-\xi_m t}\sin(\omega_m t-\phi_m), \tag{2}$$

where ξ_m, ω_m are determined from system natural frequencies and damping ratios. It can be proved that $R(t)$ can be modeled using an exact AR model (i.e. without the residual term),

$$R(t)=-\sum_{k=1}^{2M} c_k R(t-k),\quad c(z)=\prod_{m=1}^{M}\left(1-2e^{-\xi_m}\cos\omega_m z^{-1}+e^{-2\xi_m}z^{-2}\right). \tag{3}$$

However, since in real applications there is always estimation and numerical errors, the residual term is retained to account for these effects. Thus the AR model for signal ACF takes the same form as that for stationary time series. Forward covariance method will be used for AR estimation from ACF, because of the much reduced residual amplitude.

Both approaches of AR modeling will be used in this paper. In the remainder of this section, three AR-based damage features will be presented in detail.

Damage feature #1: Mahalanobis distance of AR coefficients

Mahalanobis distance is a metric to evaluate the deviation within vectorial Gaussian sample group. It is defined as:

$$D^2(\boldsymbol{\varphi}_u,\overline{\boldsymbol{\varphi}}_b)=\left(\boldsymbol{\varphi}_u^T-\overline{\boldsymbol{\varphi}}_b^T\right)\boldsymbol{\Sigma}_b^{-1}\left(\boldsymbol{\varphi}_u-\overline{\boldsymbol{\varphi}}_b\right). \tag{4}$$

where $\boldsymbol{\varphi}_u$ is the feature vector (in this case, the AR coefficients) from the unknown structural state and $\overline{\boldsymbol{\varphi}}_b$ / $\boldsymbol{\Sigma}_b$ is the mean/covariance of features from baseline state.

The current condition is determined by comparing the Mahalanobis distances computed from the baseline group of AR coefficients and those computed between the baseline group and the unknown group of AR coefficients. When the difference exceeds a certain threshold, the structural state is deemed to have changed. Often a signal needs to be segmented into many pieces so that different AR coefficient vectors may be estimated from them, thereby form a pool of enough samples for statistical processing.

Damage feature #2: Cosh distance of AR spectra

Given the AR coefficients, the corresponding spectra can be constructed:

$$S_{AR}^{(p)}(\omega)=\frac{\sigma_e^2}{\left|\boldsymbol{\varphi}\left(e^{j\omega}\right)\right|^2}=\frac{\sigma_e^2}{\left|\sum_{k=0}^{p}\varphi_k e^{-j\omega k}\right|^2}, \tag{5}$$

For feature extraction purposes model residual variance σ_e^2 is not calculated and set to 1, since its value is determined by excitation level. Cosh spectral distance based on AR spectrum estimates can be used as a frequency domain alternative to Mahalanobis distance of AR coefficients:

$$C\left(S,\,\overline{S}\right)=\frac{1}{2N}\sum_{j=1}^{N}\left[\frac{S\left(\omega_j\right)}{\overline{S}\left(\omega_j\right)}+\frac{\overline{S}\left(\omega_j\right)}{S\left(\omega_j\right)}-2\right]. \tag{6}$$

where $\bar{S}$ is the baseline spectrum, and S is the spectrum from the unknown state. The application procedure of Cosh distance is similar to that of Mahalanobis distance; a structural change is recognized through comparison between distance features within the baseline group and those from the current state group.

Damage feature #3: Q -statistic of AR residuals

Ljung and Box proposed a statistic to measure the difference between residual series and identically and independently distributed (i.i.d.) noise:

$$Q = n(n+2)\sum_{j=1}^{h}\frac{\rho_j^2}{n-j}\ . \tag{7}$$

where n is the sample size, h is the number of lags, and ρ_j is the autocorrelation at the j^{th} lag. When the structural state differs from the original, the residuals produced by filtering the new signal with the baseline AR coefficients will not resemble a white noise sequence and hence the value of Q- statistic will increase. Under the null hypothesis, Q- statistic follows a χ^2 distribution under the normality assumption of the input. But because this assumption is rarely exact, a data-driven threshold using cross-one-validation scheme will be used. Details on this threshold construction method and all feature extraction processes can be found in [5].

FEATURE PERFORMANCE VALIDATION

The features described in the previous section are applied to identify structural change for a vertical truss member in a steel truss bridge over the Allegheny River in western Pennsylvania. The bridge structure is a continuous deck truss with spans of 420 feet, 540 feet, and 420 feet, as shown in Fig. 1. The truss is 40 feet in depth and is haunched to 84 feet at the two intermediate piers. An elevation view of the bridge is shown in Fig. 1.

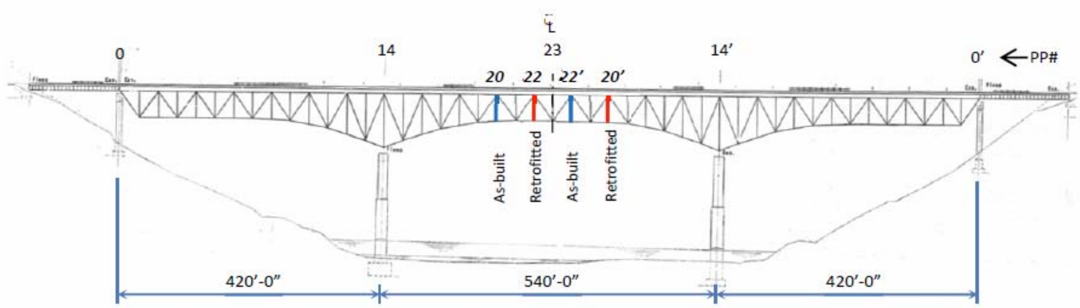

Figure 1. Side view of the truss bridge looking north (courtesy of Mr. Ian C. Hodgson)

During an inspection in June 2010, it was found that vertical members at Panel Points (PP) 20' and 22 (Fig. 1) on the north side of truss had excessive wind-induced vibration. The two members were then retrofitted by bolting a steel wide-flange member to each of them over their full height.

In a subsequent field test, transverse acceleration measurements are collected for Verticals 22 and 22'. These two members were identical before the retrofit of PP22. Each member is instrumented with eight accelerometers; four at the mid-span cross section and four at ¾ height. The damage indices will be applied to acceleration measurements from the two members in order to identify their difference. Since the indices are all extracted using single channel AR modeling, signals used here will be only from accelerometer 4, a sensor located on the inner flange tip of the middle section.

Sampling frequency for all acceleration measurements is 1000 Hz. Two datasets of 200,000 samples are collected for each member. Dataset 1, which is from the retrofitted PP22, will be used as baseline for all applications of damage detection presented in the following subsections.

Performance of AR features estimated from ambient acceleration data

The AR modeling is performed directly on ambient acceleration using Yule-Walker method, and AR order is set to 24 for all estimation processes. Each dataset is divided into segments of size 1,000 samples with no overlap, and from each segment a coefficient vector is obtained. As such, from each set of measurements a total of 200 features can be obtained. Fig. 2 shows the damage identification (in this case, characterizing the difference between the two vertical members) results from application of Mahalanobis distance and Cosh distance features. In the plot titles, Dataset 1 and Dataset 2 are from the retrofitted PP22, while Dataset 3 and Dataset 4 are from the as-built PP22'. Damage thresholds for all plots correspond to a 5% significance level, and values beyond the thresholds are marked in the plots as crosses. For Dataset 2 more false alarms are observed for the Mahalanobis distance than the Cosh distance (26 vs. 12), and in turn the former has less missed cases than the latter (i.e. points below the threshold) for Dataset 3 and Dataset 4. Thus, the feature based on AR spectra is more robust to environmental variations. For this case both features successfully identified the structural change in the form of a significant number of outliers.

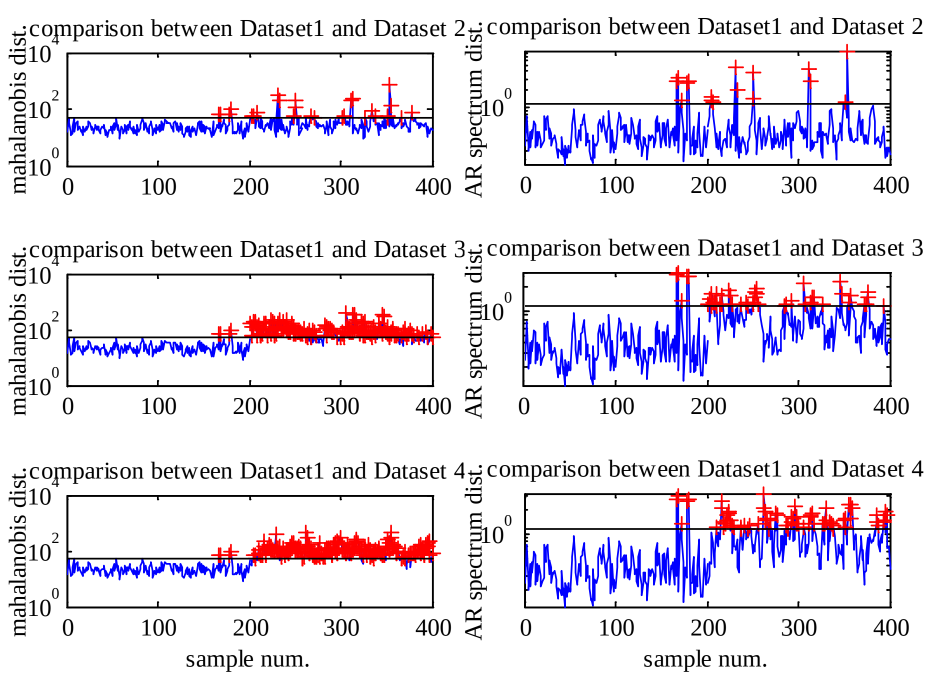

Figure 2. Mahalanobis distance and Cosh distance plots from AR estimation using ambient acceleration

The results of damage identification using the Q-statistic are contained in Fig. 3. The baseline AR coefficients are estimated using one segment from Dataset 1 and used in computation of all residual series from both retrofitted and as-built state. The lag parameter, h, is set to 60. It can be observed that this feature is not prone to false positives, as the number of outliers for both Dataset 1 and Dataset 2 is no greater than 10. As expected, when the structural state changes, much more outliers appear in the Q-statistic chart.

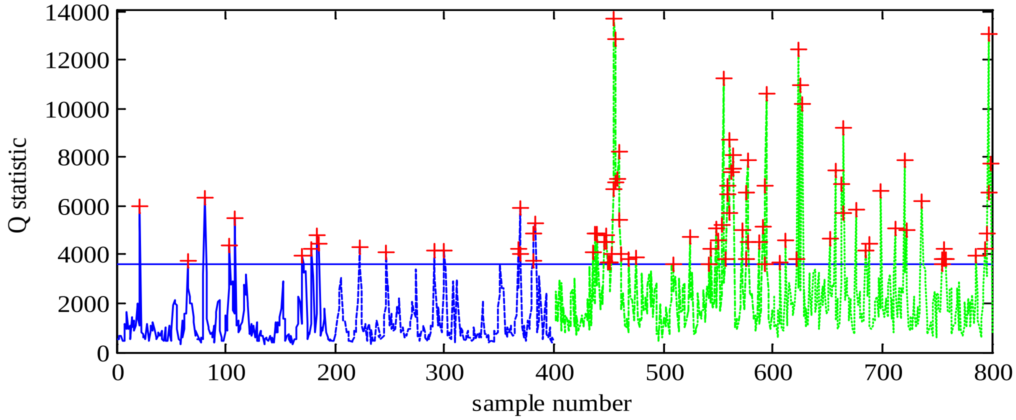

Figure 3. Q-statistic plot from AR estimation using ambient acceleration

Performance of AR features estimated from ACF of acceleration measurements

The three features are also extracted using AR estimation from signal ACF. The signal segmentation scheme, AR model order and threshold significance level adopted are the same as in the previous subsection. From each signal segment of length 1,000, an unbiased ACF estimator of length 500 is obtained starting from lag 1 and up to lag

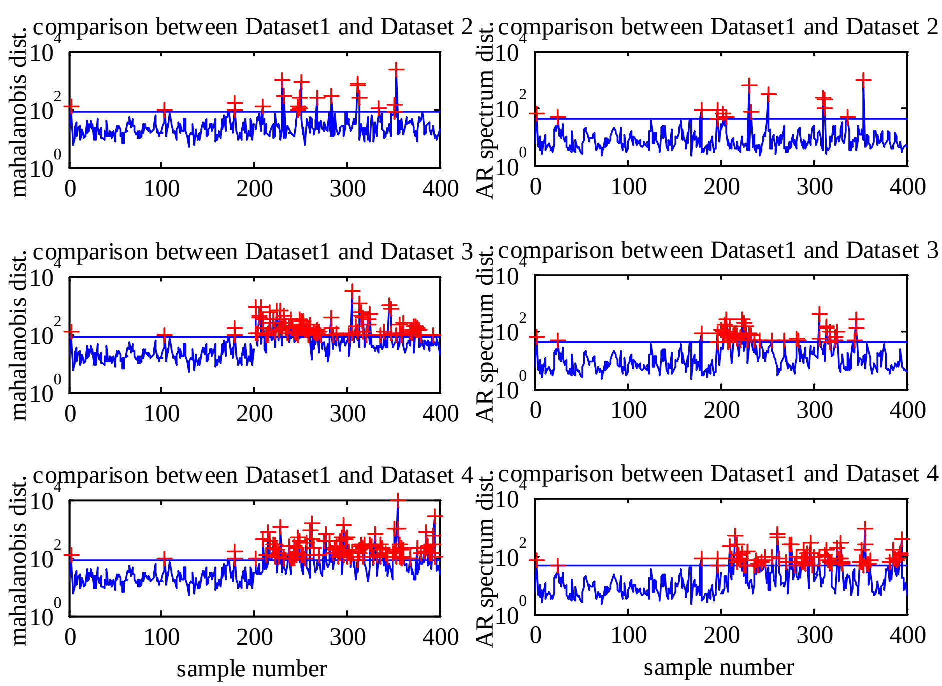

Figure 4. Mahalanobis distance and Cosh distance plots from AR estimation using signal ACF

500. From each ACF estimator an AR coefficient vector is computed. Although ideally only 2×24=48 (twice the AR order) ACF samples are needed for determination of the AR coefficients, all 500 samples are used to account for ACF estimation error and to get enough residual values for computation of the Q-statisitc, which is a function of the ACF of residuals.

Fig. 4 includes the Mahalanobis distance and Cosh distance features from ACFs of acceleration measurements. When compared with Fig. 2, it can be seen that the Cosh distance feature in this case generates slightly fewer false alarms (10 for dataset 2), yet retains the sensitivity to structural change. The Mahalanobis distance also yields more stable values for Dataset 2 (a total of 17 outliers), though the number of outliers reported for Dataset 3 and Dataset 4 is reduced. Again, both features have captured the structure change unambiguously.

Fig. 5 is the Q-statistic control chart from AR estimation based on signal ACF. The behavior of this feature here is similar to the case of direct AR modeling on vibration data (i.e. Fig. 2), except that a larger magnitude of feature values for both retrofitted state (baseline) and as-built state is observed. This is most likely because when the AR coefficients no longer 'match' the current process the residual of sinusoids exhibit strong cyclic patterns, which will make the Q-statistic value increase since the statistic is defined as a measure between a series and white noise.

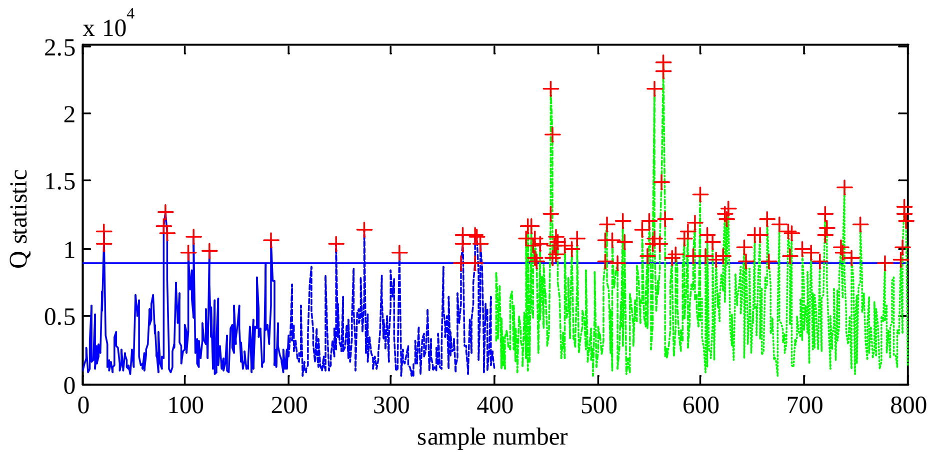

Figure 5. Q-statistic plots from AR estimation using signal ACF

CONCLUSION

Three AR damage features estimated based on the stochastic model for ambient vibration signals and deterministic model for the signal ACF are presented and employed to detect the difference between the as-built state and retrofitted state of a vertical member in a steel truss bridge. The six damage indices thus generated all successfully identified the structural change. It is observed that compared with Cosh distance and Q-statistic, the Mahalanobis distance feature is more likely to produce false-positive results.

A contrast between features from AR estimation directly on stationary time series and that estimated using the signal ACF reveals that the robustness of Mahalanobis distance and Cosh distance features improved (though not to a great extent) for the latter modeling method. This is expected because ACF computation is an averaging process and therefore should filter out some of the environmental uncertainties. Another more theoretical reason is that the AR coefficients of ACF of vibration signals of a multi-degree-of-freedom structure are functions of natural frequencies and damping properties only, which implies that the features will not be affected by changes in modal shapes (a sacrifice on the part of damage sensitivity) and also a number of non-structural-factors such as white measurement noise and excitation pattern variation. The results from Q-statistic using the two modeling methods are similar. Yet it should be noted that the tests on this bridge are performed for a relatively short time span; the merits of the ACF based modeling might become more clear for long-term monitoring of real-world structures subjected to a variety of operational conditions.

ACKNOWLEDGEMENTS

Research funding is partially provided by the National Science Foundation through Grant No. CMMI-0926898 by Sensors and Sensing Systems program, and by a grant from the Commonwealth of Pennsylvania, Department of Community and Economic Development, through the Pennsylvania Infrastructure Technology Alliance (PITA). This financial support is gratefully acknowledged. The authors would also like give thanks to Dr. Sause and Dr. Dorvash for supplying the bridge monitoring data.

REFERENCES

1. Lynch, J. P. 2007. "An overview of wireless structural health monitoring for civil structures," *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *365*(1851), 345-372.
2. Chang, P. C., Flatau, A. and Liu, S. C. 2003. Review paper: health monitoring of civil infrastructure. *Structural health monitoring*, *2*(3), 257-267.
3. Farrar, C. R. and Worden, K. 2007. "An introduction to structural health monitoring," *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, *365*(1851), 303-315.
4. Nair, K. K., Kiremidjian, A. S. and Law, K. H. 2006. "Time series-based damage detection and localization algorithm with application to the ASCE benchmark structure," *Journal of Sound and Vibration*, *291*(1), 349-368.
5. Yao, R. and Pakzad, S. N. 2012. "Autoregressive statistical pattern recognition algorithms for damage detection in civil structures," *Mechanical Systems and Signal Processing*, *31*, 355-368.
6. Zhang, Q. W. 2007. "Statistical damage identification for bridges using ambient vibration data," *Computers & structures*, *85*(7), 476-485.
7. Nigro, M., Pakzad, S.N. 2013. "The use of localized sensor networks for damage detection: a likelihood ratio test," 9th International Workshop on Structural Health Monitoring, Stanford University, CA, USA.
8. Brockwell, P. J. and Davis, R. A. 2002. *Introduction to time series and forecasting*. Springer Verlag.
9. Porat, B. 1994. *Digital processing of random signals: theory and methods*. Prentice-Hall, Inc..
10. James III, G. H., Carne, T. G. and Lauffer, J. P. 1993. *The natural excitation technique (NExT) for modal parameter extraction from operating wind turbines* (No. SAND--92-1666). Sandia National Labs., Albuquerque, NM (United States).

DES*tech* Publications, Inc.

CONTRIBUTING AUTHOR COPYRIGHT RELEASE FORM

As author of the chapter/contribution titled _*Performance comparison of different autoregressive damage features using acceleration measurements from a truss bridge*_, to appear in the *Proceedings of Structural Health Monitoring 2013,* I hereby agree to the following:

1. To grant to DEStech Publications, Inc., 439 North Duke Street, Lancaster, PA, 17602, copyright of the above named chapter/contribution (for U.S. Government employees to the extent transferable), in print, electronic, and online formats. However, the undersigned reserve the following:
a. All proprietary rights other than copyright, such as patent rights.
b. The right to use all or part of this article in future works.

DEStech Publications thereby retains full and exclusive right to publish, market, and sell this material in any and all editions, in the English language or otherwise.

1 I warrant to DEStech Publications, Inc., that I am the (an) author of the above-named chapter/contribution and that I am the (a) copyright holder of the above-named chapter/contribution granted to DEStech Publications, Inc.

2 I warrant that, where necessary and required, I have obtained written permission for the use of any and all copyrighted materials used in the above-named chapter/contribution. I understand that I am responsible for all costs of gaining written permission for use of copyrighted materials.

3 I agree to assume full liability to DEStech Publications, Inc. and its licensee, and to hold DEStech Publications, Inc. harmless for any claim or suit filed against DEStech Publications, Inc. for violation of copyrighted material used in the above-named contribution.

Please sign and date this form and retain a copy for your records. Please include original form with your chapter/paper.

Thank you for your cooperation.

Please print name: Ruigen Yao
Signed: Ruigen Yao Dated: 5/15/13

439 NORTH DUKE STREET • LANCASTER, PENNSYLVANIA 17602-4967, U.S.A. Toll Free: (866) 401-4337 • Tel: (717) 290-1660 • Fax: (717) 509-6100 E-mail: info@destechpub.com • Internet address: www.destechpub.com