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Optimal sensor configuration for flexible structures with multi-dimensional mode shapes

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Abstract

A framework for deciding the optimal sensor configuration is implemented for civil structures with multi-dimensional mode shapes, which enhances the applicability of structural health monitoring for existing structures. Optimal sensor placement (OSP) algorithms are used to determine the best sensor configuration for structures with *a priori* knowledge of modal information. The signal strength at each node is evaluated by effective independence and modified variance methods. Euclidean norm of signal strength indices associated with each node is used to expand OSP applicability into flexible structures. The number of sensors for each method is determined using the threshold for modal assurance criterion (MAC) between estimated (from a set of observations) and target mode shapes. Kriging is utilized to infer the modal estimates for unobserved locations with a weighted sum of known neighbors. A Kriging model can be expressed as a sum of linear regression and random error which is assumed as the realization of a stochastic process. This study presents the effects of Kriging parameters for the accurate estimation of mode shapes and the minimum number of sensors. The feasible ranges to satisfy MAC criteria are investigated and used to suggest the adequate searching bounds for associated parameters. The finite element model of a tall building is used to demonstrate the application of optimal sensor configuration. The dynamic modes of flexible structure at centroid are appropriately interpreted into the outermost sensor locations when OSP methods are implemented. Kriging is successfully used to interpolate the mode shapes from a set of sensors and to monitor structures associated with multi-dimensional mode shapes.

Keywords: structural health monitoring, ptimal sensor placement, optimal sensor network, wireless sensor network, odal assurance criteria

(Some figures may appear in colour only in the online journal)

Introduction

Maintaining and improving the reliability of structural systems has been the focus of structural health monitoring (SHM) research in the past two decades (Alampalli *et al* 2005, Hsieh *et al* 2006, Brownjohn 2007, Frangopol 2011). The procedures for SHM are established and methods are categorized based on their major contribution to research (Sohn *et al* 2004, Farrar and Worden 2007). In early stages of SHM, optimal sensor placement (OSP) is considered an important issue, which requires engineering judgment and experiences for decision makers. Ultimately, the optimal sensor configuration aims to perform quantitative measure for accurate damage estimation and efficient

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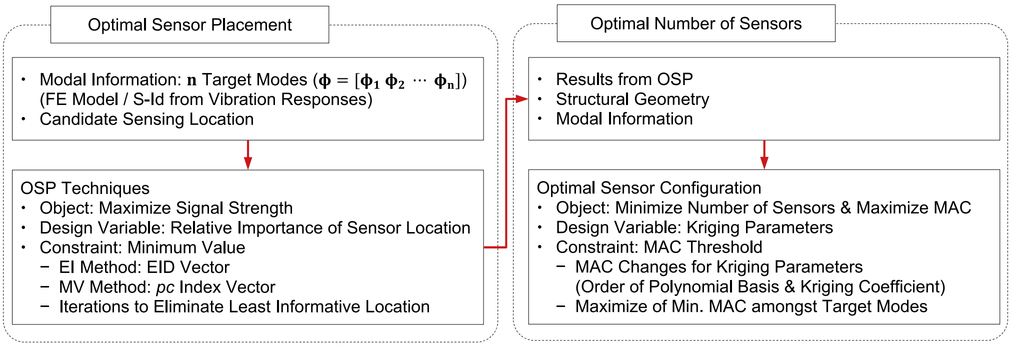


Figure 1. Framework for optimal sensor configuration.

management of sensor network systems (Kammer 1991, Yao *et al* 1992, Laory *et al* 2012). The development of smart sensor network and wireless sensing devices also prompts OSP applications that avoid redundant data processing (Spencer *et al* 2004, Lynch and Loh 2006, Pakzad 2010). The recent focus of OSP expands its applicability to damage detection, augmentation of sensors with varying sensitivities, and entire sensor configuration considering an optimal number of sensors (Kripakaran and Smith 2009, Sim *et al* 2011, Chang and Pakzad 2014a)

Most OSP techniques evaluate the signal strength at sensing locations and select the best sensor configuration that can sufficiently detect significant changes representing damage. Modal information is widely used to quantify the signal strength at each degree of freedom (DOF). For example, Kammer (1991) proposed the effective independence (EI) method which examined the error of unbiased estimator using target mode shapes. EI-driving point residue, a variation of EI proposed by Papadopoulos and Garcia (1998), uses modal frequencies in addition to target mode shapes while considering the relative contribution of each mode. Heo *et al* (1997) proposed kinetic energy (KE) which introduced modal mass for estimating KE at DOFs. The principal component analysis (PCA) is adopted to search for the best sensor configurations in the variance method which uses the covariance of target mode shapes (Meo and Zumpano 2005). In order to overcome the challenge of excessive computational costs required to investigate all possible sensor sets using the variance method, Chang and Pakzad (2014a) proposed the modified variance (MV) method in which the number of sensors for a bridge system was determined using a modal assurance criteria (MAC) (Ewins 1984) threshold.

The main objective of this study is to expand the optimal sensor configuration framework, previously developed for bridge systems (Chang and Pakzad 2014a), for flexible structures with multi-dimensional mode shapes. The framework is composed of two optimization processes: the maximization of modal information from observations and the minimization of the number of sensors. Two OSP methods (EI and MV) are investigated to quantify the signal strength and to optimize sensor locations. In order to reflect signal strength indices at candidate nodes, Euclidean norm is used. Based on the OSP results, the mode shapes for the entire structure are estimated from Kriging, a geostatistical estimator, to interpolate and extrapolate the values for unobserved locations (Matheron 1963). Kriging is a useful tool for mode shape estimation since it does not require boundary conditions and is convenient to implement for multi-dimensional mode shapes. Based on the parametric study for Kriging model, this study suggests standard conditions for general applications in mode shape estimation.

The framework is applied on a finite element model of Berks County Courthouse (BCC). The effect of Kriging parameters is investigated and the suggested conditions are compared with the optimal solution. Additionally, the performance of OSP methods is discussed for varying number of target modes.

Optimal sensor configuration

The problem of optimal sensor configuration is formulated to place sensors at the most informative locations minimizing the number of sensors and their management cost. In order to avoid a large computational burden required by investigating all possible configurations, the framework is separated into two optimization problems (Chang and Pakzad 2014a): (1) signal strength quantification for candidate sensor locations to decide the sequential priorities of sensor locations and (2) determination for the effective number of sensors to monitor target modes.

The OSP techniques are used to determine the sequential priorities of sensing locations to monitor target structural modes. The optimization aims to maximize signal strength by iteratively eliminating the least informative locations amongst the candidates. The result of OSP combined with geometry and modal information of a given structure is utilized to minimize the number of sensors and to maximize the mode shape information. Typically, these can be conflicting objectives that require searching techniques subject to unknown sensor locations. Interpolation techniques are used to estimate the mode shapes including unobserved locations based on modal information from a set of observation and the associated parameters are considered as design variables. The flowchart in figure 1 describes the entire procedure to establish optimal sensor configuration.

Two main challenges arise when the framework is applied for general civil structures. (1) Multi-dimensional mode shapes are not considered. (2) Interpolation methods such as spline or polynomial are only applicable when the boundary conditions are given such as bridge systems. This paper presents techniques to overcome these challenges while maintaining the main flow of the existing framework.

Optimal sensor placement

EI and MV methods evaluate signal strength at each DOF using target mode shapes, and are investigated in this paper. The background for these methods is briefly described in this section.

Effective independence. Kammer (1991) proposed EI method to locate sensors on space structures, based on the best unbiased linear estimator (BLUE). When a monitored structure is composed of m DOFs, and N modes are targeted, the vibration response $\mathbf{y} \in \Re^m$ is expressed as:

$$\mathbf{y} = \boldsymbol{\Phi}\mathbf{q} + \mathbf{w}. \tag{1}$$

In equation (1), $\boldsymbol{\Phi} \in \Re^{m\times N}$ is a target mode shape matrix, $\mathbf{q} \in \Re^N$ is a modal contribution factor, and $\mathbf{w} \in \Re^m$ is a corresponding stationary random noise vector. The unbiased estimator of $\mathbf{q}$ is used to evaluate fisher information matrix (FIM) which is also defined as $\mathbf{F} = \boldsymbol{\Phi}^{\mathrm{T}}\boldsymbol{\Phi}$ (Middleton 1960). In order to maximize the amount of information in FIM, EI method evaluates effective independence distribution (EID) $\mathbf{E}$ as:

$$\mathbf{E} = [\boldsymbol{\Phi}\boldsymbol{\Psi}] \otimes [\boldsymbol{\Phi}\boldsymbol{\Psi}] \cdot \lambda^{-1}\iota. \tag{2}$$

In equation (2), $\boldsymbol{\Psi}$ and λ are the eigenvector and eigenvalue matrices of $\mathbf{F}$, ι is the unity vector to sum the contribution of all target modes, and $\otimes$ represent a term-by-term matrix multiplication.

The sensor location with lowest index of $\mathbf{E}$ is discarded from the candidates and the total procedure is repeated until the most significant sensor locations are identified.

Modified variance. Chang and Pakzad (2014a) proposed MV method based on the PCA in which a set of observations is transformed into uncorrelated variables using covariance matrix of target observations. In this method, $\boldsymbol{\Phi}$ is transformed into $\boldsymbol{\Omega} = \left[\boldsymbol{\Phi} \;\; -\boldsymbol{\Phi}\right]$ to increase the rank of covariance matrix and to prevent the irregularity from the sign convention of mode shapes.

The covariance of $\boldsymbol{\Omega}^{\mathrm{T}}$ can be expressed with its sub-matrices as:

$$\mathrm{cov}\left(\boldsymbol{\Psi}^{\mathrm{T}}\right) = \begin{bmatrix} \mathbf{C}_{\alpha\alpha} & \mathbf{C}_{\alpha\beta} \\ \mathbf{C}_{\beta\alpha} & \mathbf{C}_{\beta\beta} \end{bmatrix} = \begin{bmatrix} \mathrm{cov}\left(\boldsymbol{\Psi}_{\alpha}{}^{\mathrm{T}}\right) & \mathbf{C}_{\alpha\beta} \\ \mathbf{C}_{\beta\alpha} & \mathrm{cov}\left(\boldsymbol{\Psi}_{\beta}{}^{\mathrm{T}}\right) \end{bmatrix}. \tag{3}$$

In equation (3), the subscripts $(\cdot)_\alpha$ and $(\cdot)_\beta$ denote a set of observed locations and the rest of candidate sensor locations ($\beta = m - \alpha$), respectively. The determinant of $\mathbf{C}_{\alpha\alpha}$ is maximized in order to minimize the error in unbiased estimator of $\boldsymbol{\Omega}_\beta$ (Fedorov and Hackl 1994).

The principal component index (PCI), $\mathbf{PC}$ is introduced to evaluate the signal strength at sensing locations with the consideration of the erratic dispersion in off-diagonal elements of $\mathrm{cov}\left(\boldsymbol{\Omega}_\alpha{}^{\mathrm{T}}\right)$. The ith component of $\mathbf{PC}$ is estimated by:

$$\mathbf{PC}_i = \frac{c_{ii}}{\sqrt{\sum_{j=1,\, j\neq i} c_{ij}{}^2}}. \tag{4}$$

In equation (4), c_{ii} is the ith row and jth column of $\mathrm{cov}\left(\boldsymbol{\Omega}_\alpha{}^{\mathrm{T}}\right)$.

Condensation of sensor locations. Several recent sensing systems are capable of measuring multi-dimensional vibration responses, which are used to optimize the number of physical devices. In practice it is important to optimally place a sensor network on structures that have multi-dimensional mode shapes, such as flexible buildings, circular cross section, and transform the measurements considering sensor geometry (Trochu 1993, Gu 2003, Galetto and Pralio 2010). The Euclidean norm is used to quantify the signal strength at node $\mathbf{x}_k$ such that the components in normalized EID and PCI are defined as

$$\mathbf{E}'_k = \frac{\sum_{x_i \in x_k} \mathbf{E}_i{}^2}{l_x}, \tag{5}$$

$$\mathbf{PC}'_k = \frac{\sum_{x_i \in x_k} \mathbf{PC}_i{}^2}{l_x}. \tag{6}$$

In equations (5) and (6), l_x denotes the number of component in $\mathbf{x}_k$. Each OSP method discards all associated sensors at a node, when the node is identified as the least significant sensor location.

Minimal number of sensors

In order to minimize the number of sensors to monitor the modal properties of a given structure, MAC value between target and estimated modes from the optimal sensor configuration is examined. The interpolation and extrapolation techniques based on polynomial functions require additional information and assumptions, such as a spline model to mirror the existing modal information (Ahlberg *et al* 1967). The spline models are useful in estimating the mode shapes for bridge systems where both ends of the span are normally supported. However, it cannot be implemented for building structures involving free boundary conditions at roof and multi-dimensional mode shapes. Alternatively, Kriging models (Belytschko *et al* 1994, Gu 2003) are utilized to interpolate and extrapolate the entire mode shapes in this study.

Kriging. Kriging is a geostatistical estimator which deduces the random value at unobserved points using a weighted sum of known values at neighbor locations (Matheron 1963). For mode shape estimation, Belytschko *et al* (1994) proposed the element-free Galerkin method which solves a Kriging problem based on the moving least-squares. In this method, the modal coordinates ($\boldsymbol{\Phi}_\alpha$) from a set of observation and geometric information ($\mathbf{s}$) of a given structure is used to derive the target mode matrix $\boldsymbol{\Phi}$:

$$\boldsymbol{\Phi} = f\left(\mathbf{s}, \boldsymbol{\Phi}_\alpha\right). \tag{7}$$

The best linear unbiased predictor (BLUP) of kth target mode shape is defined as a function of linear regression model such that:

$$\Phi^{\langle k \rangle}(\mathbf{x}) = \mathbf{p}(\mathbf{x})^\mathrm{T}\mathbf{a}_k + z(\mathbf{x}). \tag{8}$$

In equation (8), the superscript k in $\Phi^{\langle k \rangle}(\mathbf{x})$ indicates a modal coordinate of kth target mode at location $\mathbf{x}$; $\mathbf{a}_k$ is a coefficient vector which minimizes the error of the regression model; $z(\mathbf{x})$ is the corresponding error which is considered as a stochastic process with zero mean and non-zero covariance; $\mathbf{p}(\mathbf{x})$ is a polynomial basis associated with geometry of the structural system. For two dimensional shape functions with coordinates x and y, the basis functions are determined as:

$$\mathbf{p}(\mathbf{x}) = \begin{cases} [1] & \text{if } o_p = 0, \\ \begin{bmatrix} 1 & x & y \end{bmatrix}^\mathrm{T} & \text{if } o_p = 1, \\ \begin{bmatrix} 1 & x & x^2 & y & xy & y^2 \end{bmatrix}^\mathrm{T} & \text{if } o_p = 2. \end{cases} \tag{9}$$

In equation (9), o_p denotes the order of the polynomial basis. A Gaussian function is used to define the covariance matrix of $z(\mathbf{x})$ to consider high correlation when two locations are closely placed as:

$$R\left(\mathbf{x}_i, \mathbf{x}_j\right) = \mu \exp\left(-\theta d_{ij}{}^2\right). \tag{10}$$

In equation (10), μ is an amplification factor, θ is a correlation parameter, and d_{ij} is the relative distance between nodes $\mathbf{x}_i$ and $\mathbf{x}_j$. The BLUP problem (Sack *et al* 1989) estimates the coefficient vector $\mathbf{a}_k$ as:

$$\hat{\mathbf{a}}_k = \left(\mathbf{P}^\mathrm{T}\mathbf{R}^{-1}\mathbf{P}\right)^{-1}\mathbf{P}^\mathrm{T}\mathbf{R}^{-1}\boldsymbol{\Phi}_\alpha{}^{\langle k \rangle}. \tag{11}$$

In equation (11), $\mathbf{P} = \begin{bmatrix} \mathbf{p}(\mathbf{s}_1) & \mathbf{p}(\mathbf{s}_2) & \cdots & \mathbf{p}(\mathbf{s}_m) \end{bmatrix}^\mathrm{T}$ and the correlation matrix $\mathbf{R}$ for associated observations is defined as:

$$\mathbf{R} = \begin{bmatrix} 1 & R(\mathbf{s}_1, \mathbf{s}_2) & \cdots & R(\mathbf{s}_1, \mathbf{s}_m) \\ R(\mathbf{s}_2, \mathbf{s}_1) & 1 & \cdots & R(\mathbf{s}_2, \mathbf{s}_m) \\ \vdots & \vdots & \ddots & \vdots \\ R(\mathbf{s}_m, \mathbf{s}_1) & R(\mathbf{s}_m, \mathbf{s}_2) & \cdots & 1 \end{bmatrix}. \tag{12}$$

Accordingly, the mode shape is estimated as:

$$\Phi^{\langle k \rangle}(\mathbf{x}) = \mathbf{p}(\mathbf{x})^\mathrm{T}\hat{\mathbf{a}}_k + \mathbf{r}(\mathbf{x})^\mathrm{T}\mathbf{R}^{-1}\left(\boldsymbol{\Phi}_\alpha{}^{\langle k \rangle} - \mathbf{P}\hat{\mathbf{a}}_k\right). \tag{13}$$

In equation (13),

$$(\mathbf{x}) = \begin{bmatrix} R(\mathbf{s}_1, \mathbf{x}) & R(\mathbf{s}_2, \mathbf{x}) & \cdots & R(\mathbf{s}_m, \mathbf{x}) \end{bmatrix}^\mathrm{T}.$$

Procedure for minimum number of sensors. The optimization problem is defined to minimize the number of sensors and maximize information from the observed sensor locations. Considering that the mode shape estimation can be normalized, only two Kriging parameters are considered as design variables: order of the polynomial basis o_p and the correlation parameter θ. MAC value between targeted ($\bar{\boldsymbol{\varphi}}$) and estimated ($\tilde{\boldsymbol{\varphi}}$) mode shape vectors is used to evaluate the accuracy in modal coordinate estimation (Ewins 1984). The MAC value for kth target mode is defined as:

$$\mathrm{MAC}_k = \frac{\left|\bar{\boldsymbol{\varphi}}^\mathrm{T}\tilde{\boldsymbol{\varphi}}\right|}{\sqrt{\left(\bar{\boldsymbol{\varphi}}^\mathrm{T}\bar{\boldsymbol{\varphi}}\right)\left(\tilde{\boldsymbol{\varphi}}^\mathrm{T}\tilde{\boldsymbol{\varphi}}\right)}}, \; (k = 1, \; 2, \cdots, N). \tag{14}$$

A selected sensor configuration estimates all target mode shapes accurately, for which the MAC threshold is defined and used to search the minimum MAC amongst target modes. The purpose of MAC threshold, set to 0.95 throughout this paper, is to provide a rigorous standard for evaluating accuracy in mode shape. The design variable, order of polynomial basis and the associated variables such as the number of sensors and their locations are discretized so that the closed form solution is limited for general implementations. For these reasons, the min-max problem is formulated as:

$$\max_{\theta, o_p} \min_{k \in \boldsymbol{\Omega}} \mathrm{MAC}_k\left(\theta, o_p\right). \tag{15}$$

In equation (15), $\boldsymbol{\Omega} = \left\{k \,|\, \boldsymbol{\Phi}^{\langle k \rangle} \in \boldsymbol{\Phi}\right\}$. To solve the min-max optimization problem effectively, a searching range for θ and proper polynomial basis order are suggested in the following parametric study.

Parametric study for Kriging

Two parameters are involved for the Kriging model: (1) order of polynomial basis and (2) correlation parameter. The order of polynomial basis determines the type of Kriging, such that the zero order assumes an unknown constant for the mean of the process and is called ordinary Kriging. A positive order postulates a polynomial trend for the mean of the random process associated with geometric information, which is also known as universal Kriging (Isaaks and Srivastava 1989). The correlation parameter determines the shape of a Gaussian function such that high correlation between error terms is estimated for the closely spaced sensor locations. Based on the selections of these two associated parameters, the mode shape estimation towards target mode is affected.

A 20 story frame structure is numerically simulated to investigate the effect of Kriging parameters. The mode shapes can be categorized as torsional and transverse lateral modes of which the first seven are depicted in figure 2. The vibration response can be measured from the two corners of each floor with three sensors and used to estimate these mode shapes. Assuming that the in-plane displacement is ignorable (rigid diaphragm) and the rotational deformation is small, the mode shapes are simply represented by a combination of modal coordinates at the centroid of each floor as shown in figure 3. In order to observe structural modes, all displacements for each floor are required either at centroid $\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ or two corners $\mathbf{U} = \begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}$ which involves unidimensional and bidimensional spaces for Kriging, respectively. The modal ordinates in the $\mathbf{U}$ space can be transformed into the $\mathbf{X}$ space, for which the OSP quantifies the signal strength without unit consideration. The optimal sensor configurations are investigated for various orders and correlation parameters.

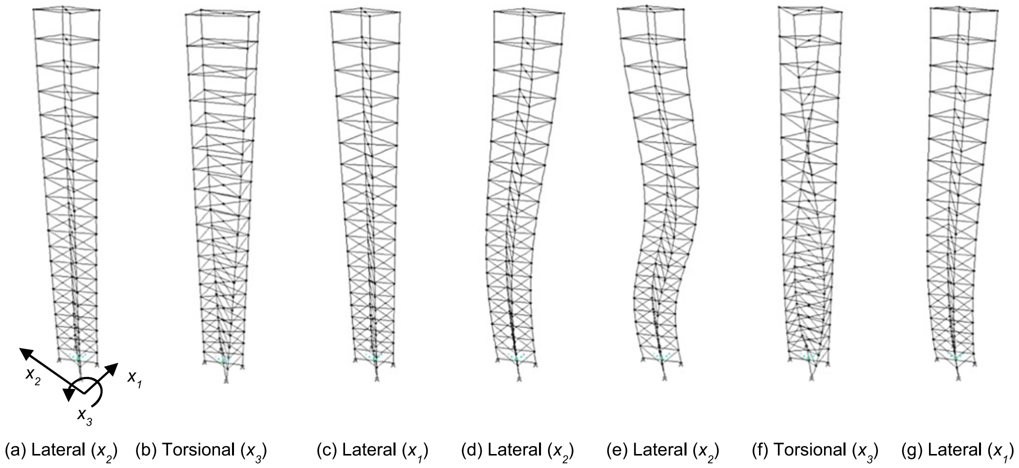
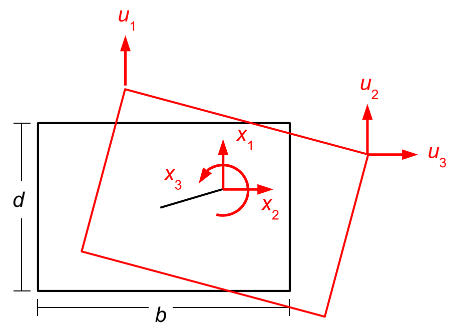


Figure 2. First seven dynamic mode shapes for 20 floor building model.



$$\mathbf{U} = \Xi\mathbf{X} \rightarrow \mathbf{X} = \Xi^{-1}\mathbf{U} \qquad \Xi^{-1} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ -d/2b & d/2b & 1 \\ -1/b & 1/b & 0 \end{bmatrix}$$

Figure 3. Transformation of mode shapes from **U** to **X** space.

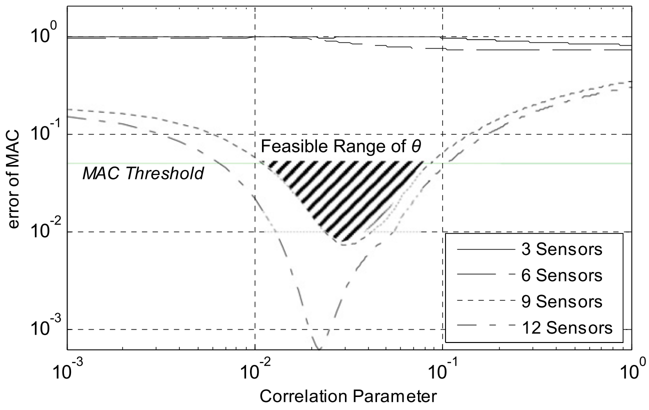


Figure 4. Feasible range of correlation parameter when seven modes are targeted.

Effects of Kriging parameters

The feasible ranges of correlation parameter, which satisfies the MAC threshold with minimum number of sensor locations, are searched to investigate the effect of Kriging parameters. It is assumed that each floor corresponds to a single candidate sensor location with three sensors.

For example, the first seven modes are targeted in **X** space and a feasible range of correlation parameter is investigated to search for the optimal number of sensors. MV is used to quantify the signal strength at candidate locations. Figure 4 shows the error in MAC value (i.e., 1-MAC) changes versus correlation parameters in logarithmic scale for various numbers of sensors (three sensors for each floor). For a MAC threshold of 0.95, the minimum number of sensors is determined as nine and the feasible range is plotted. The figure shows that the proper selection of correlation parameter affects the performance of mode shape estimation significantly.

In order to effectively provide the Kriging parameters, further investigation is conducted for a varying number of target modes. Figure 5 illustrates the feasible ranges of the correlation parameter (plotted with bar graphs) when mode shapes are expressed in **X** and **U** spaces. A dot symbol (•) used above the feasible correlation parameter range, indicates the optimal solution which maximizes MAC for each case. Regardless of space and basis order, the optimal correlation parameters are close for most cases. In this figure, the feasible ranges for quadratic basis in **U** space were not included since the result was not converging and the consistent range was not identified. The result shows that the correlation parameter has a robust relationship with the number of sensors (almost linear). The order of basis function increases the feasible ranges of correlation parameter for some cases but not significant to minimize the number of sensors. However, the increased range potentially improves the optimal sensor configuration for other structures when minimum MAC curve crosses the MAC threshold. In general, the higher orders of polynomial basis are ineffective to improve the performance of Kriging and a less than cubic order is recommended considering that the high order accompanies a heavier computational cost.

Suggested bounds for correlation parameter

The optimal solution (bullet points in figure 5) provides robust information for the upper and lower bounds for correlation parameter. For example, MV identifies three informative sensor locations at [5, 12, 20] floors with maximum

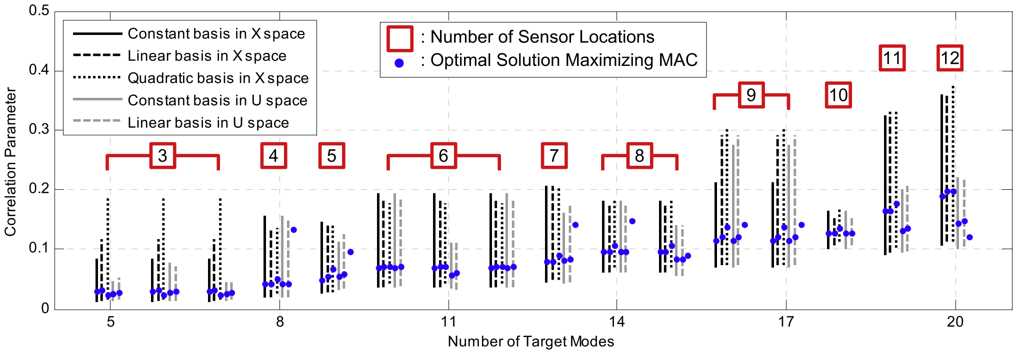


Figure 5. Feasible ranges for various target modes and the required number of sensor locations (three sensors for each location) using mode shapes in **X** and **U** spaces.

MAC of 0.95 when first seven modes are targeted. The OSP results from EI show similar performance to optimize sensor configuration and the mode shape dimension is not sensitive to Kriging parameters.

Based on the regression analysis of the previous results, the following equation for correlation parameter (θ_s) is derived as:

$$\theta_s = 0.8\frac{n_s^{\,1.25}}{L_M^{\,2}}. \tag{16}$$

In equation (16), L_M is the largest distance amongst candidate sensor locations (scale factor); n_s is the associated number of sensors. The suggested correlation parameter is applicable to interpolate mode shapes from other structural systems for the following reasons. (1) OSP methods such as EI and MV quantify signal strength effectively to estimate the target mode shapes. Previous studies showed that the sensor locations are distributed with fairly uniform spacing for simply supported beam and bridge systems and demonstrated that these are almost the same as the exact solutions (Chang and Pakzad 2014b). Similar to the simply supported beam which is the model for many bridges, shear buildings have harmonic mode shapes, suggesting that similar values for the parameter is effective for certain building structures as well. Due to the precisely quantified signal strength at candidate locations, the amount of information from a chosen sensor configuration is maximized and the geometric condition affects less than the physical number of sensors. (2) Essentially the *number of sensors* is a key factor to quantify the amount of information. (3) Other parameters such as mode shapes dimension and modal amplitudes barely affect the performance of Kriging. (4) The effect of the scale can be ignored by introducing a scale factor (L_M).

For the practical applications, the acceptable bounds have to be investigated for the selection of proper correlation parameters. Assuming that the upper and lower levels of correlation parameter have also linear relationship with the number of sensors and their coefficients are normally distributed, the sample mean and standard deviation can be used to determine the reasonable domain of correlation parameter for a specified confidence level. Equations (17) and (18) show the upper (θ_u) and lower (θ_l) bounds when the confidence level of 99.9% is applied.

$$\theta_l = 0.15\frac{n_s^{\,1.25}}{L_M^{\,2}}, \tag{17}$$

$$\theta_u = 4.52\frac{n_s^{\,1.25}}{L_M^{\,2}}. \tag{18}$$

For the optimal selection of Kriging parameter, the correlation parameter is investigated within the upper and lower bounds. The effect of these bounds is discussed in the case studies.

Case study for FE model of BCC

The proposed framework is demonstrated by investigating the optimal sensor configuration for the FE model of BCC. The building occupants complained about the excessive vibration when an aerobics class takes place on the 16th floor. In addition, the displacement of the building was noticeable during the high wind episodes and affected the comfort level of the occupants. In order to provide informative measures of the current state of the structure, the building is studied and instrumented by wired and wireless sensors (Dorvash *et al* 2014). An optimal sensor configuration is formulated here that guides the sensor placement.

Finite element model of BCC

The BCC is a16 story steel structure with four floors of parking space below the ground level, and is located in Reading, Pennsylvania. The finite element model of BCC is built in *SAP2000* as shown in figure 6. The building is mainly described as a lateral-force-resisting system which has two braced frames for each transverse direction, and masonry walls in the exteriors. Steel beams and girders are used to support concrete floors. The plan of each floor is consistent up to the 13th floor, and slender columns support the east side of upper floors.

Based on modal analysis of the FE model, 14 structural modes are identified. The first and second modes are lateral in west–east and north–south directions, respectively, and the third mode is torsional. The identified modal information for

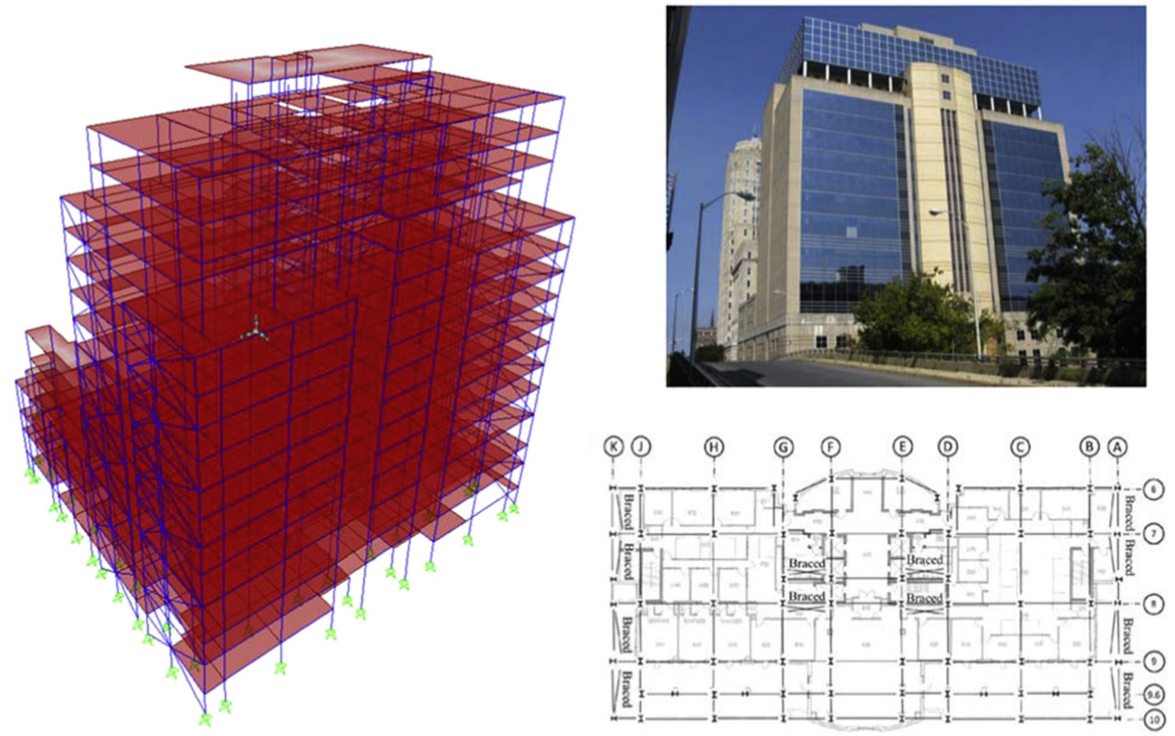


Figure 6. FE model of BCC (*SAP2000*).

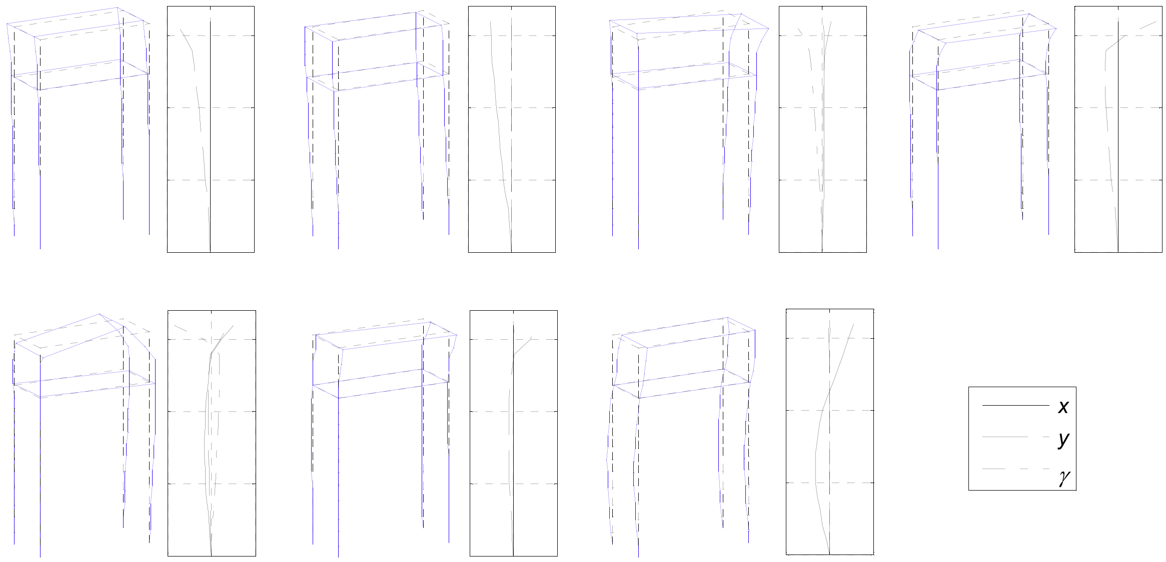


Figure 7. 3D mode shapes and the corresponding floor centroid ordinates in lateral (x and y) and rotational (γ) directions for the first seven modes.

the 14 modes is processed through OSP methods using varying number of target modes. In reality, modeling error as well as measurement noise exist, but not considered in this paper since they are not part of the scope for an OSP framework.

Optimal sensor configuration

Figure 7 illustrates the first seven mode shapes in 3D plot and corresponding modal ordinates at centroid of each floor. For this building, unlike to the numerically simulated shear model, there are modes that cannot be classified simply as lateral or torsional. For this reason, OSP methods consider mode shapes for all directions separately and the signal strength at each sensor location is normalized by the root of square sum. In general, the modal ordinates above 14th floor are relatively high due to the reduction of stiffness over the 13th floor.

Based on the framework, the optimal sensor configuration is investigated under varying number of target modes.

Table 1. Minimum number of sensors for varying number of target modes when constant, linear, quadratic, and cubic orders of polynomial basis are used.

Number of target modes	Minimum number of sensors for various orders of polynomial basis			
	Constant	Linear	Quadratic	Cubic
1	**1**	**1**	**1**	**1**
2	**1**	**1**	**1**	**1**
3	9	2	**1**	**1**
4	9	**5**	8	9
5	10	**7**	11	12
6	10	**7**	11	12
7	9	**7**	11	12
8	11	**8**	11	13
9	11	**9**	**9**	13
10	10	8	**7**	13
11	13	**11**	13	13
12	12	**11**	12	12
13	11	10	**9**	10
14	**11**	**11**	**11**	**11**

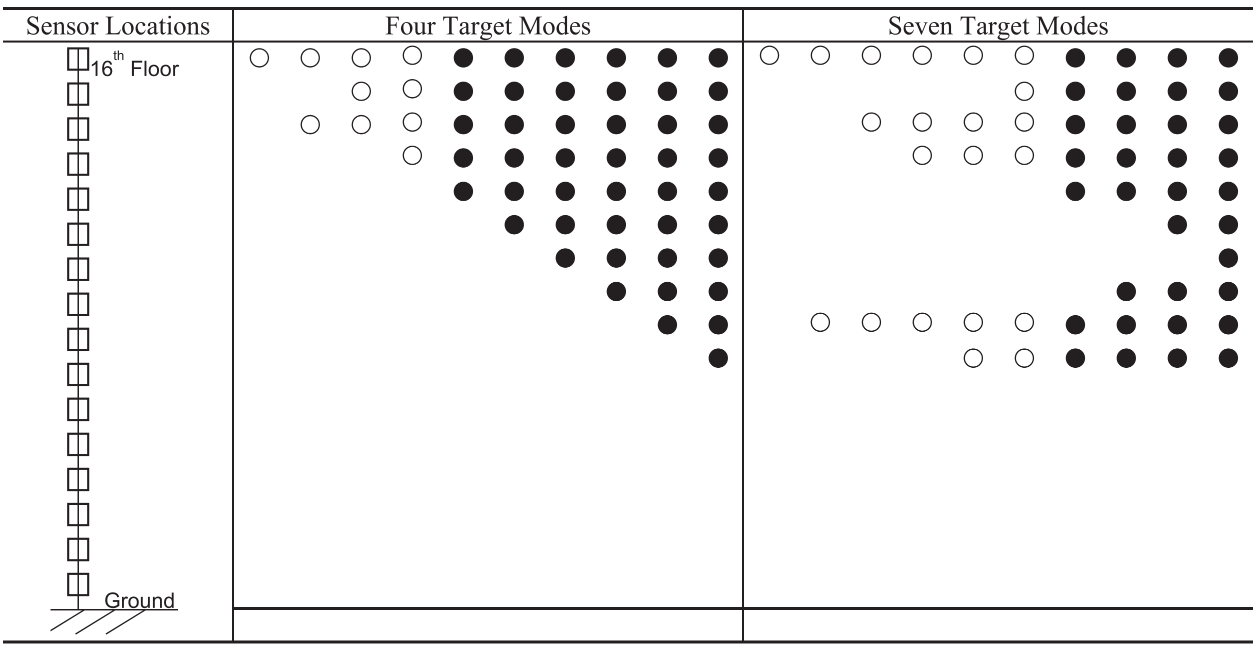
Note. Bold font indicates the optimal solution

Table 1 shows the number of sensors to satisfy the MAC criteria of 0.95 (the bold font indicates the optimal solution) when MV is used to quantify signal strength at candidate sensor locations (three sensors for each). Polynomials of up to cubic order are considered as the basis function. The higher than cubic orders are not investigated since they are over parameterized and the performance with respect to the number of sensors is negative. In general, the best performance is observed when the linear order is applied. A large sensor network is required only when the bases order is higher than linear, indicating that the higher order terms are redundant and result in negative accuracy to estimate target mode shapes.

Specific sensor configurations are investigated, which suggest the sensors should be concentrated in the top floors of the building. In many cases, the MV method identifies the informative locations from the top to the bottom, sequentially. Table 2 shows how the MV method identities the significant locations when four and seven modes are targeted, for which cases five and seven sensor locations need to be monitored (filled dots indicate that the sensor network satisfies MAC threshold of 0.95). It indicates that the sensor locations are concentrated in the top floors due to the high modal amplitudes. The increase of target modes results in signal strength increase at the middle of the building but it becomes the same when the number of sensor locations is more than ten floors. These are the same ten sensor locations identified as the most informative for quadratic and cubic orders of basis function. Consequently, the order of polynomial basis becomes important to create optimal sensor configuration.

In figure 8, the feasible ranges of correlation parameter are investigated and plotted versus the number of target modes. The correlation parameter ranges which minimize the number of sensor are plotted with bar graphs for constant, linear, and quadratic orders of basis function. The suggested upper and lower bounds, which are functions of associated number of sensors, are plotted over the correlation parameter ranges. The number on top of each target mode is the optimal number of sensor locations for each case. The large sensor networks also affect the correlation parameter ranges. In the previous example, the harmonic mode shapes can be estimated with a certain number of sensors, which is equivalent to the number of inflection points in the highest mode plus one. However, the MAC estimation is insensitive to the

Table 2. Sensor configuration comparison



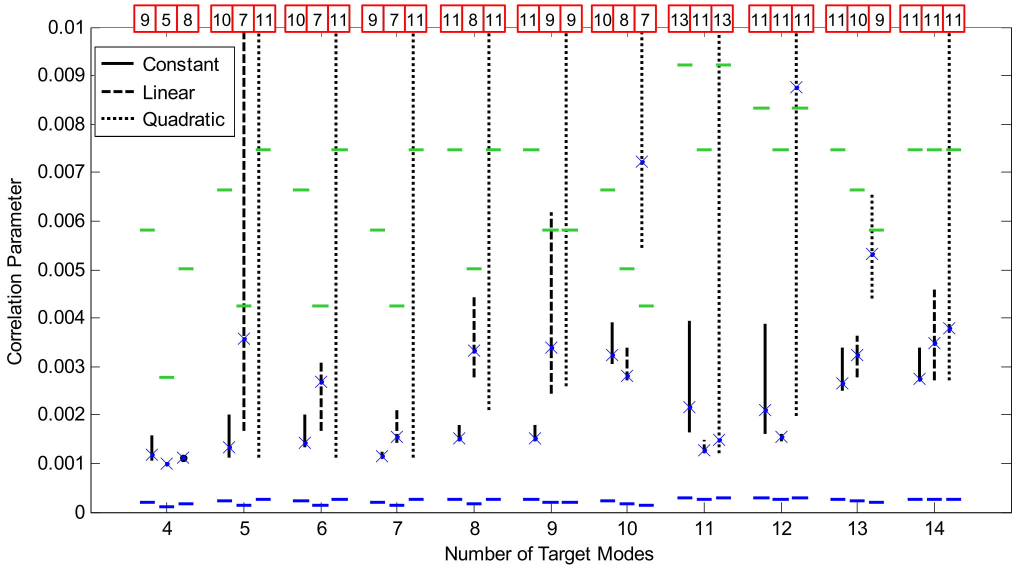


Figure 8. Feasible ranges of correlation parameters when the three orders of basis are considered and MV is used to quantify signal strength at sensor nodes (— , —: suggested upper and lower bounds; ×: optimal solution; ▢▢▢: number of sensor locations for investigated orders).

variation of correlation parameter when the large sensor network is investigated, for which the upper bound for feasible ranges is not identified. In general, the suggested bounds are suitable to identify the minimum number of sensors. Although there are cases that locate the best condition outside of the suggested bounds, these are mostly not considered as an optimal solution. When the minimum number of sensors are used, the optimal solution is searched within the suggested bounds for correlation parameters except ten target modes.

Additionally, the performance of MV and EI is compared in terms of the minimum number of sensors (figure 9). Up to five target modes, the same configurations are observed, and the rest of the results depend on the target mode information.

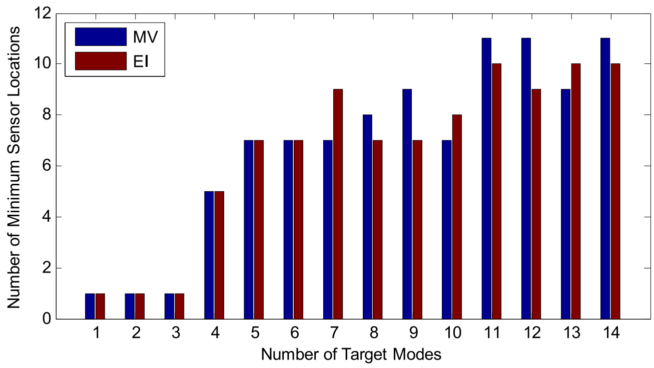


Figure 9. Comparison of the optimal number of sensors between MV and EI methods.

Conclusion

In this paper, a parametric study for Kriging is conduced and several conclusions are reached.

(1) Kriging estimates the unknown values using the sum of linear regression model and error contribution. The order of polynomial basis has to be determined for the regression model and the Gaussian function to define the correlation of error terms that vary for correlation parameters. Usually these two parameters are combined with geometric information so that the mode shapes are estimated.
(2) A torsional and two transverse modes are considered for flexible structures. The example of 20 story building model shows that the mode shapes at centroid and the outermost sensor locations of each floor result in the same optimal sensor configuration. Considering that the outermost locations are accessible for typical measurement conditions, the consistent OSP result is significant to interpret multiple types of dynamic modes. The signal strength at nodes, involved with multiple types of mode shape, is quantified using Euclidean norm of associated indices.
(3) Due to the difficulty of obtaining the optimal condition for these parameters, the parametric study was performed to derive common trends. The modal parameters from 20 story building model are simulated and the effects of these parameters are investigated for various target modes. The feasible ranges, which minimize the number of sensors with satisfying MAC threshold, are investigated.
(4) The high order of polynomial basis increases the feasible ranges for some cases. However, it potentially improves the performance of Kriging when more complex mode shapes are considered. The result shown in the example indicates that the linear order of basis function performs better in terms of the number of sensors. Considering that the order increase accompanies high computational cost, low order is generally recommended.

(5) The optimal correlation parameter has an almost linear relationship with the number of associated sensors. Based on the regression analysis, upper and lower searching bounds are suggested and their performance is demonstrated by an example.

Although the Kriging shows promising performance in estimating the mode shapes from a set of sensor configuration, it needs further efforts for practical implementations.

(1) More practical studies are needed to verify the performance of the method.
(2) The order of the basis function affects the optimal sensor configuration. For the multi-dimensional mode shapes, the linear order is the most effective. However, further investigation is required to demonstrate the relationship between the type of modes and the order of the basis.

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References

Ahlberg J H, Nilson E N and Walsh J L 1967 *The Theory of Splines and Their Applications* (New York: Academic)
Alampalli S, Ettouney M M and Agrawal A K 2005 Structural health monitoring for bridge maintenance *Bridge Struct.* **1** 345–54
Belytschko T, Lu Y Y and Gu L 1994 Element-free Galerkin methods *Int. J. Numer. Methods Eng.* **37** 229–56
Brownjohn J M 2007 Structural health monitoring of civil infrastructure *Phil. Trans. R. Soc.* A **365** 589–622
Chang M and Pakzad S N 2014a Optimal sensor placement for modal identification of bridge systems considering number of sensing nodes *J. Bridge Eng.* **19** 04014019
Chang M and Pakzad S N 2014b A parameter optimization approach for mode shapes estimation using Kriging interpolation *Proc. Int. Modal Analysis Conf.* (Bethel, CT: Society for Experimental Mechanics)
Dorvash S, Pakzad S, Naito C, Hodgson I and Yen B 2014 Application of state-of-the-art in measurement and data analysis techniques for vibration evaluation of a tall building *Struct. Infrastruct. Eng.* **10** 654–69
Ewins D J 1984 *Modal Testing: Theory and Practice* (Letchworth: Research Studies Press)
Farrar C R and Worden K 2007 An introduction to structural health monitoring *Phil. Trans. R. Soc.* A **365** 57–82
Fedorov V V and Hackl P 1994 Optimal experimental design: spatial sampling *Calcutta Stat. Assoc. Bull.* **44** 57–82
Frangopol D M 2011 Life-cycle performance, management, and optimisation of structural systems under uncertainty: accomplishments and challenges *Struct. Infrastruct. Eng.* **7** 389–413
Galetto M and Pralio B 2010 Optimal sensor positioning for large scale metrology applications *Precis. Eng.* **34** 563–77
Gu L 2003 Moving Kriging interpolation and element-free Galerkin method *Int. J. Numer. Methods Eng.* **56** 1–11
Heo G, Wang M L and Satpathi D 1997 Optimal transducer placement for health monitoring of long span bridge *Soil Dyn. Earthq. Eng.* **16** 495–502
Hsieh K H, Halling M W and Barr P J 2006 Overview of vibrational structural health monitoring with representative case studies *J. Bridge Eng.* **11** 707–15
Isaaks E H and Srivastava R M 1989 *Applied Geostatistics* (Oxford: Oxford University Press)
Kammer D C 1991 Sensor placement for on-orbit modal identification and correlation of large space structures *J. Guidance, Control, Dyn.* **14** 251–9
Kripakaran P and Smith I F 2009 Configuring and enhancing measurement systems for damage identification *Adv. Eng. Inform.* **23** 424–32
Laory I, Hadj Ali N B, Trinh T N and Smith I F 2012 Measurement system configuration for damage identification of continuously monitored structures *J. Bridge Eng.* **17** 857–66
Lynch J P and Loh K J 2006 A summary review of wireless sensors and sensor networks for structural health monitoring *Shock Vib. Dig.* **38** 91–130
Matheron G 1963 Principles of geostatistics *Econ.Geol.* **58** 1246–66
Meo M and Zumpano G 2005 On the optimal sensor placement techniques for a bridge structure *Eng. Struct.* **27** 1488–97
Middleton D 1960 *An Introduction to Statistical Communication Theory* (New York: McGraw-Hill)
Pakzad S N 2010 Development and deployment of large scale wireless sensor network on a long-span bridge *Smart Struct. Syst.* **6** 525–43
Papadopoulos M and Garcia E 1998 Sensor placement methodologies for dynamic testing *AIAA J.* **36** 256–63
Sacks J, Susannah S B and Welch W J 1989 Design for computer experiments *Technometrics* **31** 41–7
Sim S H, Carbonell-Márquez J F, Spencer B F and Jo H 2011 Decentralized random decrement technique for efficient data aggregation and system identification in wireless smart sensor networks *Probab. Eng. Mech.* **26** 81–91
Sohn H, Farrar C R, Hemez F M, Czarnecki J J, Shunk D D, Stinemates D W and Nadler B R 2004 A review of structural health monitoring literature: 1996–2001 *Report Number* LA-13976-MS (Los Alamos, NM: Los Alamos National Laboratory)
Spencer B F, Ruiz-Sandoval M E and Kurata N 2004 Smart sensing technology: opportunities and challenges *Struct. Control Health Monit.* **11** 349–68
Trochu F 1993 A contouring program based on dual Kriging interpolation *Eng. Comput.* **9** 160–77
Yao L, Sethares W A and Kammer D C 1992 Sensor placement for on-orbit modal identification of large space structure via a genetic algorithm *IEEE Int. Conf. on Systems Engineering (Kobe, Japan)* pp 332–5