Multisensor Aggregation Algorithms for Structural Damage Diagnosis Based on a Substructure Concept

Ruigen Yao and Shamim N. Pakzad

Abstract: One of the important goals of structural health monitoring is damage detection. Although many methods have been proposed to detect the existence of structural damage, relatively few studies are found on higher-level damage diagnosis such as identification of the location and extent of damage. In this paper, multiple substructural damage identification models based on regression between internal responses and boundary responses of individual beam elements in either plane or three-dimensional space are derived. Three damage indexes are defined from regression model characteristics, and two change-point analysis methods are adopted to capture changes in damage index sequences which are extracted from structural monitoring data sets from healthy and unknown states. Possible damage locations are identified as where the most significant changes in the damage indexes occur, and a voting scheme is used to synthesize the results from different algorithms. This damage detection approach is straightforward and efficient, with the regression coefficients directly related to the structural stiffness properties. The numerical and experimental application results show that the method successfully identifies and locates structural change in most of the cases.

DOI: 10.1061/(ASCE)EM.1943-7889.0000879. © 2014 American Society of Civil Engineers.

Author keywords: Structural health monitoring (SHM); Substructures; Regression analysis; Structural analysis; Stochastic models; Steel bridges.

Introduction

Damage detection and assessment is very important for timely and proper maintenance of civil infrastructures. Traditional practices mostly rely on human inspections, and as a result the cost of maintenance is high and the period of maintenance is long. With the development of digital sensing technologies, data-driven structural health monitoring (SHM) (Chang et al. 2003) has been proposed as an attractive alternative because of the low cost and premise of scalability.

The general methodology of data-driven SHM consists of the design and deployment of a sensing network on the structure and data collection, selection, and analysis. The declining price of sensing systems has made the implementation of dense sensor networks feasible, and computationally efficient and practically effective algorithms need to be devised accordingly to process the large amount of data produced through monitoring systems. The aim is to form an online automatic structural assessment procedure that requires little involvement of technicians/experts, thereby reducing the cost and bias in the decision-making process.

System identification/modal realization is the classical method used for SHM purposes, and many studies have investigated the functions of eigenfrequencies and eigenvectors estimated from structural responses as damage indicators (Pothisiri and Hjelmstad 2003; Hassiotis and Jeong 1995; Salawu 1997). Although these damage features are theoretically well-grounded for understanding, it may take a lot of time and resources for some algorithms to achieve good results, and more importantly, these features are found to be insensitive to local damage (Doebling et al. 1998). Also, model updating for the monitored system will often be needed to perform high-level damage detection (Pothisiri and Hjelmstad 2003; Weber and Paulet 2010). In an effort to overcome these problems, damage detection techniques that adopt statistical analysis on structural responses (Zhang 2007; Sohn and Farrar 2001) have been proposed. These features are found to be sensitive to damage and also to the excitation condition change as a result of the inherent information limitation for this family of methods.

To strike a balance between a method’s damage sensitivity and performance robustness, several approaches that treat the entire structural system as an assembly of substructures and model each substructure independently have been developed and tested (Gul and Catbas 2011; Hernandez-Garcia et al. 2010; Law and Yong 2011; Lei et al. 2013; Yan et al. 2013; Yao and Pakzad 2014; Yuen and Kataygiotis 2006). This family of algorithms often uses the response from within the substructure for output and the responses at the substructural boundaries for input, and therefore damage features based on the estimated substructural model parameters and model residuals should convey information strongly related to the physical behavior of the substructures. Law and Yong (2011) proposed a substructural model updating technique using recursive sensitivity-based state-space model modification. Lei et al. (2013) introduced another system state-space model modification method from the recursive application of Karman filters for local damage detection in both small- and large-scale simulated structures. Yuen and Kataygiotis (2006) presented a substructural identification approach using maximum likelihood estimation and hypothesis testing based on the Bayesian probability distribution. Gul and Catbas (2011) applied autoregressive exogenous (ARX) input modeling to measurements collected from clusters of sensors for damage localization in a numerical mass-spring system and a steel grid setup in a laboratory. Yan et al. (2013) used artificial neural
network training to model the dynamics of substructures within mass-spring and multistory systems, and the standard F-test on the model fit ratio to decide whether or not damage occurred for a substructure. Hernandez-Garcia et al. (2010) proposed a structural decomposition approach that uses Chebyshev series expansion to model the mass-normalized interstory restoration force signal and then extracted the structural property information. Two regression-based methods that directly use acceleration responses to evaluate substructural stiffness-mass ratios in a decentralized manner have been developed (Yao and Pakzad 2014). Compared with the aforementioned approaches, substructural methods are generally better at damage location detection, less affected by operational condition variations than scalar signal analysis, and less computationally demanding than many global system identification approaches. Substructural physics and global structural mechanical properties are also interrelated. Yun and Bahng (2000) showed that natural frequencies and mode shapes can be adopted as input to neural networks for substructural model updating. Barroso and Rodriguez (2004) introduced a transformational relationship between the estimated modal properties and the mass-normalized interstory stiffness for multistory structures.

Among the previously listed studies on substructural damage identification, several use black-box models such as the numerical ARX model and neural network that cannot be directly related to structural physics. For those that explicitly address the damage-induced structural stiffness loss, the algorithms are either devised mainly for structures with linear topology (i.e., mass-spring and shear building systems) or involve complex iterative operations (for example, Kalman filtering). Moreover, in many studies only numerical examples are presented for algorithm validation purposes. To assess the local stiffness variation for structures with more complex geometry in an efficient manner, two regression-based damage identification methods based on substructural beam element modeling in plane and space are proposed in this paper. Two types of element models are developed: one considers only the static effects and the other takes into account the dynamic effects by assuming an intermediate concentrated mass. The regression expression for each model can be formed in either the time or frequency domain. Damage features based on these regression models are then integrated with change-point analysis (CPA) methods (Brodsky and Darkhovsky 1993; Nigro et al. 2014) and voting schemes for structural damage assessment. The effectiveness of the proposed methodology is validated through numerical simulations and experimental implementations on laboratory and field structures.

This paper is organized as follows. First, the development of substructural beam models for both the two-dimensional (2D) and three-dimensional (3D) cases is presented. Second, the regression procedure for the substructural models and three damage features is formulated. Third, a voting-based information fusion process used to combine the damage detection results from different models/damage features/CPA techniques is detailed. Fourth, data from a simulated truss are used to demonstrate the proposed 3D modeling methods. Fifth, the implementation results on a laboratory frame and a real-world truss bridge for the algorithms are presented, and their damage identification and localization performances are compared and contrasted. Finally, conclusions are drawn on the merits and demerits of the methods.

**Substructural Model Development Based on the Finite-Element Concept**

To apply the linear regression techniques for substructural damage identification, an input-output linear model for the substructures under investigation needs to be clarified. The response of a substructure is controlled by two types of inputs: those acting directly on the substructure itself and those acting on its boundary. The relationship between these influences and the substructure response reflects certain physical properties of the substructure. When the structural responses collected are structural vibration (the most commonly monitored signal in SHM applications), the relevant structural physical properties are the material constitutive relationships.

The displacement response at an internal node of a substructure can be expressed as the following function:

$$
\mathbf{r}_i = h(\mathbf{S}, p, \mathbf{x}_i)
$$

where $\mathbf{S}$ = boundary force; $p$ = excitation acting on the substructure; and $\mathbf{x}_i = \text{coordinate vector of the internal node}$. Here, $h(\cdot)$ is determined from both the substructural constitutive relationships and the geometry, while $S$ is a function of the boundary displacement response ($\mathbf{r}_b$) and its higher-order derivatives with respect to the system coordinates ($\mathbf{r}_b^{(n)}$)

$$
S = g(\mathbf{r}_b, \mathbf{r}_b^{(n)}).
$$

Exact characterization of a substructural system requires continuous measurement of system behavior along the boundaries, which is impractical because most of the existing sensing systems are discrete and sparse. Therefore, it is assumed that the boundary responses have a finite number of degrees of freedom and can be safely inferred from measurements collected at a few boundary locations. On the other hand, external excitation on the substructure generally cannot be measured directly; however, there are cases where it can be expressed using other measurable quantities. For example, if the excitation is the inertial force, then it is a function of substructural system mass and acceleration [i.e., $p = p(m, \mathbf{r}_b, \mathbf{r}_b)$].

When the structure is linear and the substitution principle can be applied, Eq. (1) becomes

$$
\mathbf{r}_i = h(\mathbf{S}, \mathbf{x}_i) + h(p, \mathbf{x}_i) = h\left(g(\mathbf{r}_b, \mathbf{r}_b^{(n)}), \mathbf{x}_i\right)
$$

(static case with no directly applied excitation)

$$
\mathbf{r}_i = h(\mathbf{S}, \mathbf{x}_i) + h(p, \mathbf{x}_i) = h\left[g(\mathbf{r}_b, \mathbf{r}_b^{(n)}), \mathbf{x}_i\right] + h[p(m, \mathbf{r}_b, \mathbf{r}_b), \mathbf{x}_i]
$$

(dynamic case with no directly applied excitation)

Here, the velocity measurements are not considered because damping assumed to be negligible, and the $\mathbf{r}_b/\mathbf{r}_b^{(n)}$ terms are referring to the data collected at discrete locations. With both the internal and boundary responses of the substructure sampled, a model reflecting the underlying structural physics can be identified using existing numerical techniques. The accuracy of the results will depend on how closely the measurements represent the actual substructural input and response. The whole idea bears some similarity to the substructuring method adopted in finite-element (FE) modeling; only the latter aims at solving for structural response by assembling substructural stiffness matrices into one global stiffness matrix and forming equilibrium equations, while the former estimates the structural stiffness properties from a known structural response and operates on individual substructures.

The remainder of this section will be devoted to construction of linear substructural models for beam elements. In the first
substitution, substructural models for beam elements in planar space are presented and the associated input-output relationships are defined. The relationships are then generalized for modeling of beam members in 3D Euclidean space in the second subsection. The 2D and 3D substructural beam elements introduced here are modeled independently; the interaction effect is taken into account by the measurements made at both ends of each beam element. This treatment is valid as long as the Euler-Bernoulli beam model is a good approximation of the real/simulated beam portions.

**Substructural Models for Beams in Plane**

Static and dynamic models for beams in plane will be formed in this subsection in accordance with Eq. (3).

### Static Beam Model in Plane

For a continuous Euler-Bernoulli beam with section stiffness $EI(x)$ and no intermediate load [Fig. 1(a)], displacement $u(x)$ will satisfy the fourth-order differential equation

$$\frac{d^4[EI(x)u(x)]}{dx^4} = 0 \tag{4}$$

Only the real part of the frequency domain representation will be retained for regression because it contains more power than the imaginary part. For all derivations herein, subscript $\text{Cst}$ is a regression constant. The physical interpretation of the coefficients of the function variables is shown in Fig. 1(b). The coefficients are numbered in the order in which their corresponding variables appear inside the function brackets in Eqs. (6) and (7), i.e., $a_C = \beta_1 a_A + \beta_2 \dot{e}_A + \beta_3 a_B + \beta_4 \dot{e}_B$. In the plot in Fig. 1(b) it is assumed that a strain gauge is applied to the top surface of the beam element. Here, $h_A$ and $h_B$ are used to denote the vertical distances from the section centroid at A and B to their respective strain gauge locations. This knowledge will be useful in the understanding of the substructural behavior, and in light of possible structural damage, the identification of the damage location and extent. It can be seen that the coefficients of the acceleration terms ($\beta_2$ and $\beta_4$) will not be affected by substructural stiffness loss unless a hinge formed in the

Constants for the homogenous solution can be determined using the displacement and slope angles at both beam ends. When the beam section has a linear stress-strain relationship, the conditions of the slope can be replaced with conditions of strain (at the upper/lower surface)

$$u(x) = f_1(u_A, \epsilon_A, u_B, \epsilon_B) \tag{5}$$

where $f_1(\cdot)$ = certain functional relationship. When the solution to Eq. (4) is linear with respect to its homogeneous constants, $f_1$ becomes a linear function. In vibration monitoring applications that measure the system’s acceleration, this model can still be applied by taking the second derivative of Eq. (5) with respect to time if the system stiffness-to-mass ratio is large

$$a_C = \ddot{u}(x) = f_1(\ddot{u}_A, \ddot{e}_A, \ddot{u}_B, \ddot{e}_B) = f_1(a_A, \dot{e}_A, a_B, \dot{e}_B) \tag{6}$$

where $a_A$, $a_B$, and $a_C$ = acceleration signals measured at Positions A, B, and C (Fig. 1). This relationship can also be formulated in the frequency domain by taking the one-sided Fourier transform of Eq. (6)

Only the real part of the frequency domain representation will be retained for regression because it contains more power than the imaginary part. For all derivations herein, subscript $\text{Cst}$ is a regression constant. The physical interpretation of the coefficients of the function variables is shown in Fig. 1(b). The coefficients are numbered in the order in which their corresponding variables appear inside the function brackets in Eqs. (6) and (7), i.e., $a_C = \beta_1 a_A + \beta_2 \dot{e}_A + \beta_3 a_B + \beta_4 \dot{e}_B$. In the plot in Fig. 1(b) it is assumed that a strain gauge is applied to the top surface of the beam element. Here, $h_A$ and $h_B$ are used to denote the vertical distances from the section centroid at A and B to their respective strain gauge locations. This knowledge will be useful in the understanding of the substructural behavior, and in light of possible structural damage, the identification of the damage location and extent. It can be seen that the coefficients of the acceleration terms ($\beta_2$ and $\beta_4$) will not be affected by substructural stiffness loss unless a hinge formed in the

### Diagrams

Fig. 1. (a) Deflected shape and free-body diagram of the static beam element model; (b) coefficients of each variable when the static model function is linear (variables before/after the hyphen are the associated output/input variables)
beam. However, because this static model is rarely an exact description of real beam members, these coefficients are still retained for structural state evaluation.

Beam Model in Plane with a Lumped Mass

The model introduced in the previous subsection addresses only static/quasi-static applications. Here, a model that incorporates a part of the dynamic effects is constructed by adding a lumped mass on the beam [Fig. 2(a)]. The following is obtained by applying the generalized force concept:

\[
\hat{u}_C(i\omega) = f_2(u_A, \varepsilon_A, u_C, \varepsilon_C, u_B, \varepsilon_B) \approx \hat{u}_A(i\omega), \hat{\varepsilon}_A(i\omega), \hat{u}_B(i\omega), \hat{\varepsilon}_B(i\omega)
\]

\[
= f_2 \left( \frac{\hat{u}_A(i\omega) + \hat{\varepsilon}_A(i\omega)}{-\omega^2}, \frac{\hat{u}_C(i\omega) + \hat{\varepsilon}_C(i\omega)}{-\omega^2}, \frac{\hat{u}_B(i\omega) + \hat{\varepsilon}_B(i\omega)}{-\omega^2} \right)
\]

\[
\text{Re}[\hat{u}_C(i\omega)] = \text{Re}\left( f_2 \left( \frac{\hat{u}_A(i\omega)}{-\omega^2}, \frac{\hat{\varepsilon}_A(i\omega)}{-\omega^2}, \frac{\hat{u}_C(i\omega)}{-\omega^2}, \frac{\hat{\varepsilon}_C(i\omega)}{-\omega^2} \right) \right) + Cst_2
\]

Fig. 2(b) illustrates the physical interpretation associated with each variable coefficient (\(a_C = \beta_1 m_A + \beta_2 \varepsilon_A + \beta_3 u_C + \beta_4 \varepsilon_C + \beta_5 u_B + \beta_6 \varepsilon_B\), provided that all regressors are mean removed). Note that here each coefficient is represented with a force instead of a variable before/after the hyphen are the associated output/input variables).

Substructural Models for Beams in 3D space

Substructural modeling for beams in 3D space is innately a more complex problem, especially when torsional deformation is taken into consideration. In this subsection, the relationships between the internal and boundary responses of an arbitrary 3D beam (Fig. 3) will be derived.

Static Beam Model in 3D Space

The homogeneous governing differential equations for the translation (at the centroid) and rotation (with respect to the shear center) of a linear beam in 3D space are examined (without considering second-order effects) in the following equations (Seaburg and Carter 1997):

\[
\frac{d^2 [EJ_x(z)dz^2 u(z)]}{dz^4} = 0
\]

\[
\frac{d^2 [EJ_y(z)dz^2 v(z)]}{dz^4} = 0
\]

\[
\frac{d[GK_T(z)d\phi(z)]}{dz^4} = \frac{d[EJ_\theta(z)d^3\phi(z)]}{dz^4} = 0
\]

where \(u\), \(v\), and \(\phi = x\)-direction translation, \(y\)-direction translation, and torsion angle at the section centroid, respectively; \(EI_x\) and \(EI_y\) = section bending stiffness about the \(x\)- and \(y\)-axes, respectively;

\[
u(x) = f(u_A, \varepsilon_A, m\varepsilon_C, u_B, \varepsilon_B)
\]

When \(f\) = linear function (conditions for this to hold are still the same as in the previous part), Eq. (8) can be reformulated as

\[
a_C = \hat{u}_C = f_2(u_A, \varepsilon_A, u_C, \varepsilon_C, u_B, \varepsilon_B)
\]

where \(f_2\) = different linear function. Its corresponding frequency domain representation is


where \( \mathbf{e}_A \) and \( \mathbf{e}_B \) = vectors consisting of strain measured at three different locations along the perimeter of Sections A and B, respectively (Fig. 3). The torsional component accounts for all the coupling between the translational movements in the \( x \)- and \( y \)-directions, and thus the translational acceleration responses collected from an intermediate location on the beam surface can be expressed using the following functions:

\[
\ddot{u}_C = f_4' (\ddot{u}_A, \ddot{u}_B, \ddot{\phi}_A, \ddot{\phi}_B, \ddot{\mathbf{e}}_A, \ddot{\mathbf{e}}_B) = f_4' (\ddot{u}_{A1}, \ddot{u}_{B1}, \ddot{u}_{A2}, \ddot{u}_{B2}, \ddot{\mathbf{e}}_A, \ddot{\mathbf{e}}_B)
\]

\[
\ddot{v}_C = f_5' (\ddot{v}_A, \ddot{v}_B, \ddot{\phi}_A, \ddot{\phi}_B, \ddot{\mathbf{e}}_A, \ddot{\mathbf{e}}_B) = f_5' (\ddot{v}_{A1}, \ddot{v}_{B1}, \ddot{v}_{A3}, \ddot{v}_{B3}, \ddot{\mathbf{e}}_A, \ddot{\mathbf{e}}_B)
\]

\[
\mathbf{e}_A = [\mathbf{e}_{A1}, \mathbf{e}_{A2}, \mathbf{e}_{A3}], \quad \mathbf{e}_B = [\mathbf{e}_{B1}, \mathbf{e}_{B2}, \mathbf{e}_{B3}]
\]

(13)

Upper case letters in the variable subscripts are the section labels and the numbers following these letters denote a point along the section (Fig. 3). Assuming \( f_4 \) and \( f_5 \) are linear in coefficients, the corresponding frequency domain representations of the expressions are

\[
\mathbf{r}(z) = [u(z), v(z), \phi(z)]^T = f_3(\mathbf{r}_A, \mathbf{r}_B, \mathbf{e}_A, \mathbf{e}_B) \quad (12)
\]

In the 3D case, the relationship of the variable coefficients to the structural stiffness quantities is more complicated and it is especially difficult to directly present the physical equivalents of the coefficients of the strain terms. Fig. 4(a) contains a schematic illustration on the physical interpretations of the coefficients associated with the acceleration regressors when the complete strain measurements are available. For clarity, only a surface is drawn instead of a 3D beam.

**Lumped Mass Beam Model in 3D Space**

The development of a lumped mass beam model in 3D space is similar to that in 2D space. The assumption made here is that the total inertia effect of the substructure could be accounted for by using the accelerations measured at an intermediate point on the beam. The functions are thus derived as

\[
\ddot{u}_C = f_6' (u_A, u_B, u_{C1}, \phi_A, \phi_B, \mathbf{e}_A, \mathbf{e}_B) = f_6' (u_{A1}, u_{B1}, u_{A2}, u_{B2}, u_{C1}, \mathbf{e}_A, \mathbf{e}_B)
\]

\[
\ddot{v}_C = f_7' (v_A, v_B, v_{C1}, \phi_A, \phi_B, \mathbf{e}_A, \mathbf{e}_B) = f_7' (v_{A1}, v_{B1}, v_{A3}, v_{B3}, v_{C1}, \mathbf{e}_A, \mathbf{e}_B)
\]

(15)

and their frequency domain counterparts are

\[
\ddot{u}_C (i\omega) = f_6' (\ddot{u}_{A1}, \ddot{u}_{B1}, \ddot{u}_{A2}, \ddot{u}_{B2}, \mathbf{e}_A, \mathbf{e}_B) \frac{\text{linear}}{\text{linear}} \Rightarrow f_6' (\ddot{u}_{A1}, \ddot{u}_{B1}, \ddot{u}_{A2}, \ddot{u}_{B2}, \mathbf{e}_A, \mathbf{e}_B)
\]

\[
\ddot{v}_C (i\omega) = f_7' (\ddot{v}_{A1}, \ddot{v}_{B1}, \ddot{v}_{A3}, \ddot{v}_{B3}, \mathbf{e}_A, \mathbf{e}_B) \frac{\text{linear}}{\text{linear}} \Rightarrow f_7' (\ddot{v}_{A1}, \ddot{v}_{B1}, \ddot{v}_{A3}, \ddot{v}_{B3}, \ddot{v}_{C1}, \mathbf{e}_A, \mathbf{e}_B)
\]

(16)
Figs. 4(b and c) contain schematic plots showing the physical meaning of the coefficients related to the acceleration variables. The lumped mass model is a simplified approximation of the real behavior. However, to make this approximation more accurate the model needs to incorporate more variables, and then may suffer from overparameterization. This problem will become evident when the whole structure is not very well excited (i.e., the response only contains a few dynamic modes).

When the substructure is subjected to the ambient/white noise load, the models described in this section will be applicable by using the correlation of the signals with the regressand signal as a free-decay response (James et al. 1993). The corresponding frequency domain relationship will then be defined for the auto/cross-power spectral densities instead of the Fourier transform.

It should be noted that, similar to full structures, substructures need to have appropriately defined boundary conditions to ensure stability. Depending on the location of the beam element within a structure, the beam models can have different boundary conditions including the physical boundary condition and nodal response measurements. When the beam element is connected to the external support of the global structure on one side, the interface behavior at this support side is not measured and the element boundary condition is the same as the global boundary condition at this support (e.g., fixed, partially fixed, etc.). Otherwise, the boundary conditions can be characterized by the complete set of acceleration and strain measurements at beam element end, i.e., the beam can be treated as simply supported at both ends with the support translation described by acceleration and the applied beam end moments reflected by strain. In other words, unknown sensing channels can be replaced with appropriate known boundary constraints, i.e., the translation constraint for acceleration in the same direction and the rotational constraint for affected strain measurements. Boundary conditions do not appear in the regression model, leading to a reduction in the size of the regressor matrix. In such circumstances, the element is no longer a simply supported beam and physical interpretation of the coefficients should be evaluated with the constraints in consideration.

#### Table 1. Linear Regression Models Derived from Substructural Beam Models

<table>
<thead>
<tr>
<th>Model type</th>
<th>Choice of regressor/regressand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Static beam (time domain)</td>
<td>$Y = \mathbf{a}_C(t_j), \quad X = [\mathbf{a}_0(t_j), \Delta^2 { \mathbf{E}_0(t_j) }]$</td>
</tr>
<tr>
<td>Static beam (frequency domain)</td>
<td>$Y = \text{Re} { \tilde{\mathbf{a}}_C(i\omega_j) / \omega_j }, \quad X = \text{Re} \left{ \begin{bmatrix} \tilde{\mathbf{a}}_0(i\omega_j) / \omega_j, &amp; -\omega_j \tilde{\mathbf{E}}_0(i\omega_j) \end{bmatrix} \right}$</td>
</tr>
<tr>
<td>Beam with lumped mass (time domain)</td>
<td>$Y = \mathbf{a}_C(t_j), \quad X = \left[ \int a_0(t_j) \mathbf{E}_0(t_j), \int \mathbf{a}_C(t_j) \right]$</td>
</tr>
</tbody>
</table>
In the cases where the measurement set is incomplete and no substituting boundary constraints can be found for the missing signals (such as for the second experimental case study introduced subsequently in this paper), the interaction dynamics cannot be accurately captured by the substructural model, and the modeling error on the whole would increase. Note that because only dynamic displacement can be inferred from acceleration, the derived substructural beam models take into account only the vibrational behavior of the beam, and the strain measurements need to be deducted by the mean to have its constant component removed.

### Damage Feature Extraction from Substructural Regression Model Formulations

The general multiple-input/single-output linear regression problem can be formulated as

\[ Y = X\beta + \epsilon \]  \hspace{1cm} (17)

where \( Y \) = \( n \times 1 \) regressand vector; \( X \) = \( n \times m \) regressor matrix; \( \beta \) = \( m \times 1 \) regression coefficient vector; and \( \epsilon \) = \( n \times 1 \) residual series. In the size definitions, \( n \) denotes the number of observations, and \( m \) denotes the number of input series. The selected regressand/regressors for the various substructural models presented in the previous section are summarized in Table 1. From the definition of regression, there should be at least one regressor variable for coefficient estimation. Here, the size of the regressor matrix will vary depending on the substructural boundary measurements/constraints. (Please refer to the final paragraph in the previous section.) Note that for all of the regression schemes, only acceleration and strain signals are used because they are the most commonly measured vibrational responses. In Table 1, subscript \( j = 1, \ldots, N \) is a range variable, which represents the collection of sample points in the time/frequency domain depending on the situation. Here, \( a_j = [a_{j1}, a_{j2}, \ldots, a_{jN}] \) and \( e_j = [e_{j1}, e_{j2}, \ldots, e_{jN}] \) are vectors of boundary acceleration and strain measured at a series of discrete locations, with \( d \) and \( d' \) being the number of acceleration and strain sensing points, respectively. Here, \( \Delta^2 \{ \cdot \} \) represents the central difference of a signal; \( \Delta \{ \cdot \} \) denotes the reconstructed displacement from the acceleration signal inside the brackets by applying the conventional finite impulse response filter described in Hong et al. (2010); and the macroaccent, \( \hat{\cdot} \), denotes the detrended signal. The purpose of detrending is to eliminate the regression constant from the regression models, which is associated with the system static deformations in the time domain signal and the vibration initial conditions in the one-sided frequency spectrum. In the applications herein, a two-sided spectrum is adopted instead of a one-sided spectrum because it is easier to estimate the former from the data. The estimated regression coefficients \([\hat{\beta} = (XX')^{-1}X'Y]\) from the least-squares method, the ratio of the variance of regression residuals from the baseline model to that of the signal \([RF1 = \text{var}(\epsilon_{BL})/\text{var}(Y) ; \epsilon_{BL} = Y - X\hat{\beta}_{BL}]\), and the ratio of the residual variance from the baseline model to that from the current state model \([RF2 = \text{var}(\epsilon_{CS})/\text{var}(\epsilon_{BL}) ; \epsilon_{CS} = Y - X\hat{\beta}_{CS}]\) will be used as the damage features.

### Statistical Information Synthesis for Reliable Damage Prognosis

In the previous section, the regression-based damage indexes were presented. Two CPA methods, one based on the maximum cumulative sum and one based on the minimum mean-square error (Brodsky and Darkhovsky 1993; Good 1999; Nigro et al. 2014), will be used to identify the point at which a statistically significant change occurs in damage feature sequences extracted from chronologically arranged data sets representing the structural baseline and unknown states. Thereby, each substructural damage identification/localization algorithm can be formed from three parts: (1) regression model construction, (2) damage index extraction from regression, and (3) decision making using CPA on all damage index sequences. For the first part, four choices are available for either the 2D or 3D beam element (Table 1). For the second part, three indexes are presented as regression coefficients and two functions of regression residuals. For the third part, two CPA methods are available. As such, the number of all possible combinatorial damage detection algorithms is \(4 \times 3 \times 2 = 24\). In applications, it is found that the algorithms rarely yield unanimous results, which is caused by different levels of sensitivity to measurement and modeling inaccuracies and possible overparameterization. In light of these limitations, a triple-layer voting scheme will be used to pool the damage identification/localization results from all algorithms for an accurate decision on the current structural state.

Majority voting (Jain et al. 2000) is used for damage existence recognition. Each proposed classification algorithm can generate two types of errors: false alarms (Type I) and missed cases (Type II). Because no prior information on Type I/II error probability is

![Fig. 5. Illustration of the calculation of the two significance of change indicators: (a) normalized damage indication variable (NDIV); (b) normalized mean shift (NMS) \([\text{std}(\bullet) = \text{SD} \text{ of the bracketed sequence}]\)
available for the algorithms, it is assumed that the errors are of equal occurrence likelihood. As such, simple majority voting (i.e., half of the votes are needed for a decision) among the 24 combinatory substructural damage identification algorithms is used to detect damage existence because this decision system does not discriminate against either type of error. The median of all captured change points (implying the time when damage occurred) is selected as the nominal change point in accordance with the median voter theorem, which states that a majority voting system will select the outcome most supported by the median voter (Congleton 2002).

The voting strategy for damage location is a more complex problem and requires more preliminary preparation. Possible damage locations are found as positions of those substructures with most significant changes in damage features produced via different algorithms. To quantify the significance of variations from CPA, two indicators are introduced as shown in Fig. 5: the normalized damage indication variable (NDIV) and the normalized mean shift (NMS). The function of the NDIV against the change-point number has a shape with a unit-magnitude peak at the exact/real change point and raised cosine roll-off at two sides. Thus, the value of this significance factor depends on the difference between the detected change-point number and the real change-point number: the larger the difference, the smaller the NDIV value. The NMS is basically the ratio of $\Delta \mu$, which is the difference between the means of the feature values before and after the exact change point over the average of the SD of features calculated from the prechange samples and post-change samples $[\text{SD}(\text{FC1}) + \text{SD}(\text{FC2})]/2$. The two indicators are based partly on engineering judgment and partly on the definition of the damage indexes: NDIV represents a scaled version of the a posteriori probability of the actual change point given the detected change point, and the asymmetric raised cosine window is chosen based on the Monte Carlo simulation results of the change-point probability distribution; NMS measures the relative mean shift after the change/damage, and the SD is used for normalization because it reflects both the magnitude of the damage index value and the reliability of the estimate.

Note that because the exact change-point location is not known a priori, the nominal change point from the second stage of voting is used instead. The NDIV indicators from the algorithms with different CPA methods but the same other components will be summed up because NMS is not affected by the CPA methods and there is no reason to give one indicator more weight than the other. Again, in the spirit of equal weight voting, the NDIV and NMS values from the same substructural models and either of the residual-based features

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**Fig. 6.** Information synthesis from different algorithms based on the voting concept

**Fig. 7.** (a) Snapshot of the truss in ABAQUS 6.13 (the damaged portion of the middle vertical member is in the lower part and marked by a bold line); (b) sections being monitored along the vertical; (c) acceleration sensor labels per each section

are added up, such that the coefficient- and residual-based features will exert the same amount of influence in the final decision. These measures are taken because voting works best for independent observations; however, these pooled terms tend to be highly correlated.

The two damage location indicators together with the two categories of feature extraction methods and four substructural models give 16 ways to predict possible damage location(s) that will serve as candidates for the third round of voting. Tie-breaking procedures can be devised for multiple damage locations suggested by coefficients from different substructures by examining the significance of change for other coefficients in their respective substructures. For residual-based features, the weight will be divided evenly among locations that tie for the most significant change (i.e., 1/2 weight if there are two candidates, 1/3 weight if there are three, etc.). There is also a subtle difference in the handling of locations indicated via coefficients between the 2D and 3D substructural approaches: for the former the voting weight is evenly split between the locations of regressand and regressor nodes (i.e., 1/4 weight for each regressor node if there are two and 1/2 weight if there is only one regressor), while for the latter only the regressand node location is considered because of the more complex boundary conditions and dynamics. To better illustrate the procedure, Fig. 6 includes a flowchart on the process of damage detection and characterization using voting. The three stages of voting are clearly marked by a shaded background.

Numerical Validation of the Damage Detection Algorithms

In this section, the proposed damage detection methodology will be applied to a numerical space truss model for evaluation of the substructural models for space beam elements. This simulation helps to support the effectiveness of the proposed 3D substructural approach because the strain data collected in the experimental example introduced in the subsequent section is incomplete (i.e., there are less than three strain measuring points at each free section).

Case Study: Steel Truss Structure Simulated in ABAQUS

To evaluate the effectiveness of the proposed approach, a truss structure was simulated in ABAQUS using its space beam elements [Fig. 7(a)]. Damage is introduced in one of its vertical members by assigning a smaller section to a portion of the component between Sections 5 and 6 [Fig. 7(b)]. The replacement section has a 16.7% narrower flange width and 25% thinner web thickness. Concentrated random translational force in both directions and torsional moment excitation were applied at 1/6 height, midheight, and 5/6 height of the member, respectively. For each structural scenario, five data sets were simulated at a sampling rate of 1,000 Hz from seven sections. Each data set lasted 19 s. The sensing plan for each section is given in Fig. 7(c). Because ABAQUS does not directly provide strain/translation information for specific points on a beam element section, sectional moments (i.e., bending moments around the two axes and torsion moment) were used to calculate the strain, and the section rotational and centroidal translational accelerations were used to compute the acceleration measurements at the sensing points. Five percent white noise was added to the signals to simulate measurement noise.

Damage Identification—Change-Point Histograms

The previously outlined algorithms correctly reported the existence of damage. They were applied to all beam substructures that can be formed based on the sensing locations. When applying frequency domain techniques, only samples larger than the median response were used for noise-robust performance. The values extracted from five sets of baseline signals were compared with those from five sets of signals collected from the damaged state. Thus, the ideal case is that all damage indexes that report damage through CPA show a change point at 6. However, in the results acquired from applying the damage localization algorithm to the data, the histogram of change points (Fig. 8) had a wide spread, and the correct change-point location needed to be recognized by taking the median. The errors were caused both by model error and noise from the sensing measurements. One way to counter the interference of large noise variance is to collect more vibration signals in order to have more estimated damage feature samples for the CPA.

Damage Localization—Identification of the Location Where the Most Significant Change Occurs

Table 2 contains the damage locations identified from the algorithms. Basically, the sensor location(s) that corresponds to the largest change in damage location indicator values (either NDIV or NMS of the damage indexes, as calculated from the equations in Fig. 5) are identified as the damage location. Notice here that tie-breaking did not completely eliminate multiple choices for the NDIV-coefficient method. The numbering of sensing locations in Table 2 is a combination of section and sensing point labels on each section [Figs. 7(b and c)]. The algorithms here very accurately predict the damage location.

In accordance with the previously outlined procedure, a direct voting scheme among the results from different types of methods is

![Fig. 8. Change-point histograms for the truss simulation example](image-url)
used to decide on the most probable location of damage. The ranking results of the possible damage locations are included in Table 3. It should be noted that only those locations with more than one vote are listed. All of the ranked damage locations are around the real damaged portion.

**Damage Characterization through Examining the Coefficient Changes**

Fig. 9 displays plots of coefficients from the regression models (listed in Table 1) with maximum NMS at two ranked damage locations, 54 and 61. Only those coefficients associated with acceleration regressors are included. In the title for each plot, the number before the hyphen is the regressand and that after the hyphen is the regressor. The bracketed arrow points to the mean shift direction after the change point. The shaded backgrounds indicate a statistically significant change for the coefficient sequences, and the dashed lines represent the average values of the coefficients before/after the change point. For the substructural modeling results at Node 54, Coefficient 54-54 has an increase in its absolute value as a result of structural change, suggesting that the effect of mass loss on this model outweighs the effect from stiffness loss. The absolute values of Coefficients 54-43 and 54-44 increase after damage, implying strengthened correlation between responses at Sections 4 and 5. The values of Coefficient 54-64 drop below zero, which is because the substructural model is not an exact representation of the simulation and also because the model, which uses 10 regressors, can be overparameterized if the structure is not well excited over a wide range of modes. For the substructural model with Sensing Point 61 as the regressand node, the regression coefficient of responses from Node 61 on those from Node 72 (Fig. 9, Plot 61–72) is always zero because responses at Nodes 71 and 72 are negatively correlated as a result of Section 7 only having torsional responses. Among the two sets of identified coefficients, Coefficient 61-71 shows an increasing trend in its absolute value while 61-52 shows a decrease. This indicates that the responses at Node 61 become more dependent on responses from Node 71 and less on those from Node 52 after the structural change. Thus, the overall behavior of the coefficient sets that have been identified by CPA evinces a loss of correlation between responses at Sections 5 and 6 in both translational directions, implying that stiffness reduction for the portion between Sections 5 and 6 around both the strong and weak axes. In the subsequent validation examples the results will all be presented in the same order of damage identification/localization/characterization; however, the subheadings will be omitted to preserve the flow of the paper.

![Fig. 9. Acceleration-related coefficients of the regression models: (a) Model 3 with Regressand Node 54; (b) Model 4 with Regressand Node 61](image)

**Experimental Validation of the Damage Detection Algorithms**

To further test the accuracy of the algorithms, they were applied to detect and locate damage in a real 2-bay frame specimen in the laboratory and structural change in a steel truss bridge using vibration measurements collected from these structures. The 2-bay frame was used to authenticate the algorithms based on an in-plane substructural beam element. The truss bridge members had vibration in the 3D space, and thus served as a test bed for the algorithms based on the substructural beam model in 3D space.

**Case 1: Planar Steel Frame Specimen Tested in the Laboratory**

Three different damage scenarios were set up on the 2-bay frame laboratory specimen constructed from steel tubes [Fig. 10(a)]. The frame was instrumented with 21 accelerometers and nine strain gauges, and an electrodynamic shaker was mounted to its left corner to provide random excitations during testing. Three damage scenarios...
were simulated on the frame by replacing a certain member from the intact structure with another tube that had 22% reduced wall thickness, and 40, 40, and 24 data sets were collected when simulating the first, second, and third damage scenarios, respectively. For each scenario, half of the data sets were from the damaged case and half were from the undamaged case. The sampling frequency was set at 500 Hz and the sampling duration for each data set was 10 s. The sensing and damage schemes are illustrated in Fig. 10(b).

Because only a limited number of sensing locations had strain gauges, the results on damage identification/localization using the proposed algorithms were only obtained for the beam substructures with strain gauges at their ends. It can be seen from Fig. 11 that the identified change points for the three damage scenarios all have a relatively dispersed distribution. This is especially evident for the third damage scenario case, where there are fewer available data sets. Still, the median change-point values coincide with the real change points.

The damage locations predicted by algorithms based on the various models and/or various damage indexes are listed in Table 4. It is noticed that while for the third damage state the damage location is very clearly indicated by almost all algorithms, for the other two damage states the algorithms suggest quite a few distinct locations (with the correct damage location among them), thus making direct determination of the damage location difficult. This is because the replacement of the switch-out portions to simulate damage tends to affect the whole frame (the frame needs to be effectively disassembled and assembled again). To select the most probable damage locations, the voting scheme was applied to pick the top three sites with the highest votes (Fig. 12). Three sites were chosen because theoretically that is the largest number of locations that can be affected by a single damage. The results agreed well with the actual damage locations, with the latter identified as either the first or second choice from voting in each damage scenario. Therefore, it is concluded that regression models based on beam elements are

![Fig. 10. (a) Two-bay steel frame; (b) frame drawing with strain gauge and accelerometer locations](image)

![Fig. 11. Histograms of the change-point locations for the three damage scenarios created in the frame experiment (the median change point for each case is indicated as the tall thin bar)](image)

<p>| Table 4. Damage Locations as Determined from Multiple Algorithms for the Frame Experiment |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|</p>
<table>
<thead>
<tr>
<th>Damage location indicator category</th>
<th>Damage Scenario 1 (at Location 6)</th>
<th>Damage Scenario 2 (at Location 15)</th>
<th>Damage Scenario 3 (at Location 17)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDIV from regression coefficients</td>
<td>16-17-18</td>
<td>16-17-18</td>
<td>8-9</td>
</tr>
<tr>
<td>NMS from regression coefficients</td>
<td>8-7</td>
<td>8-7</td>
<td>8-9</td>
</tr>
<tr>
<td>NDIV from RF1 and RF2</td>
<td>8/20</td>
<td>8/17/20</td>
<td>6/8/17/20</td>
</tr>
<tr>
<td>NMS from RF1 and RF2</td>
<td>8</td>
<td>8</td>
<td>6</td>
</tr>
</tbody>
</table>

Note: When two numbers are hyphenated, the first number denotes the regressand node; when three numbers are hyphenated, the second number denotes the regressand node.
effective in damage localization applications for a steel frame structure.

The model coefficients that reported the most significant change at the suggested damage locations from the voting scheme are plotted in Fig. 13. The usage of the shaded background, dashed line, and parenthetical arrows is the same as in Fig. 9, and the suffix s represents coefficients associated with strain. For the first damage case, the regression coefficients between the acceleration channels demonstrated significant change, which resulted from ignoring the system mass and nonlinearity brought by bolt connections. On the other hand, the absolute values of the coefficients of both strain regressors shifted toward zero after damage, signaling a stiffness reduction that can be attributed to a connection loosening. The analysis of the coefficients from beam element modeling about Location 6 also revealed a similar trend. Note here that the coefficients pertaining to strain had small magnitudes because the data

---

**Fig. 12.** Contrast of real damage locations on the frame and suggested locations from the voting scheme in the experiment

**Fig. 13.** Plots of selected model coefficients extracted from data sets from the frame experiment: (a) Model 2, Damage State 1; (b) Model 3, Damage State 2; (c) Model 2, Damage State 3

**Fig. 14.** Elevation view of the truss bridge looking north (courtesy of Mr. Ian C. Hodgson)
were recorded in microstrains. For the second damage scenario, Coefficient 15-16s showed a significant decrease, suggesting that a stiffness reduction occurred between Locations 15 and 16. For the third damage scenario, again the coefficients from the acceleration regressors reported changes as a result of model inexactness. Coefficient 17-18s had a significant drop in its values for the last six data sets, indicating section stiffness loss.

**Case 2: Members of a Steel Truss Bridge under Ambient Conditions**

The damage detection algorithms based on substructural modeling for the beam in 3D space were applied to identify structural change for a vertical truss member in a steel truss bridge over the Allegheny River in western Pennsylvania. The bridge structure is a continuous deck truss with spans of 128, 164.6, and 128 m, as shown in Fig. 14. The truss is 12.2 m in depth and is haunched to 84 ft at the two intermediate piers.

During an inspection in June 2010, it was found that the vertical members at panel points (PPs) 20’ and 22 (Fig. 14) on the north side of truss had excessive wind-induced vibration. The two members were then retrofitted by bolting a steel wide-flange member to the web of each of them over their full height. In a subsequent field test, ambient vibration measurements were collected for vertical PPs 22 and 22’. These two members were identical before the retrofit of PP 22. Each member was instrumented with four accelerometers at the midspan cross section and another four at the 3/4 height cross section [Figs. 15(a and b)]. Also, at each end of each member two accelerometers and one strain gauge were mounted [Fig. 15(c and d)]. Five data sets were collected at a frequency of 1,000 Hz for 100 s, respectively, from retrofitted member PP 22 and unretrofitted PP 22’. The damage indexes were applied to these measurements to identify the difference between these two members.

A preliminary examination of the collected data revealed that the accelerometers mounted at the top of vertical PP 22 were dysfunctional, and thus these two channels could not be incorporated in the substructural models. Therefore, it was assumed that the translations were not included in the models. The histograms of the change points from (a) models excluding the lower cord translations and (b) models including those translations (the median change point for each case is indicated as the tall thin bar) are shown in Fig. 16.

**Table 5. Regression Models Formed for 3D Beam Substructures Found in PP 22/PP 22’**

<table>
<thead>
<tr>
<th>Substructure location</th>
<th>Translation direction</th>
<th>Regressor signala</th>
<th>Regressand signalb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between the top gusset and 3/4 height cross section</td>
<td>Longitudinal</td>
<td>A5, A6, (SG1-SG2), SG3</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>Transverse</td>
<td>A7, A8, SG1, SG2, SG3</td>
<td>A3</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>A4</td>
</tr>
<tr>
<td>Between the midheight cross section and the bottom gusset</td>
<td>Longitudinal</td>
<td>A1, A2, (SG1-SG2), SG4, A11b</td>
<td>A5</td>
</tr>
<tr>
<td></td>
<td>Transverse</td>
<td>A3, A4, SG1, SG2, SG4, A12b</td>
<td>A7</td>
</tr>
</tbody>
</table>

*aFor signal names, A = acceleration; SG = strain measurements.

*bOptional.*
The numbering of these locations coincides with the numbering of the accelerometers in Fig. 15.

In this experiment only two strain gauges were instrumented at the midsection and one at the top/bottom of the members. These were not theoretically sufficient for beam element modeling in 3D space, which requires three distinct channels of strain measurements at each end of the element. Assuming the gusset plate completely restrains rotation around the weak axis, additional independent boundary conditions are still needed for a solution to the substructural problem. However, because no further data were available from the tests on this bridge, the regression models were constructed from the incomplete data in this case. Table 5 presents a list of the regressand/regressor pairs used for substructural modeling—based damage detection in this case. Two substructures were formed from measurements made at four levels of the vertical member, and four input-output formulations were established for each substructure. There are several points meriting clarification regarding determination of the regressor and regression channels given in Table 5: (1) because strain measurements were not available at the lower interface of the upper substructure (the 3/4 height cross section), strain measurements within the substructure (i.e., at the midspan of vertical member) were used. This substitution is plausible because construction of a linear regression model for substructures implicitly assumes that responses at any point within the substructure is a linear combination of certain boundary responses, and thereby the boundary responses can be replaced by the same number of noncorrelated responses within the substructure—one drawback of this approach is that physical interpretations of the regression coefficients become more obscure; (2) because the strain at Location SG2 cannot be induced by bending around the weak axis and torsion of an I-section, here the difference of measurements at SG1 and SG2 is used as a regressor for models associated with transverse vibration in order to avoid redundancy in parameterization; and (3) for the lower substructure the translational measurements at the bottom can either be included or excluded in the regression models for structural change identification. Here, both options are explored and their results will be presented and contrasted in the rest of this section.

Histograms of the identified change points from the damage detection algorithms are displayed in Fig. 16. Regardless of whether the lower cord acceleration is included or not, the graph peaks at six, which is the correct change point. To obtain further details on this structural change, the physical locations that report the largest change in the various damage indexes, together with their voting results, are summarized in Tables 6 and 7. Here, all locations are presented because there are only two overlapping substructures, and one damage location can very well affect all nodes. The locations obtained from the models, including and neglecting the lower cord acceleration, are similar; however, in the former case the changes in the lower substructure (represented by Location 5-8) are more prominent (Table 7), which is a result of more accurate modeling. For both types of modeling, regression based on longitudinal vibration tends to report more significant change than that based on transverse vibration, indicating that section stiffness properties about the weak axis are affected relatively more than that about the strong axis. Fig. 17 shows the coefficients from the regression models with maximum NMS at the top two ranking damage locations (namely, Locations 5 and 8). As in the ABAQUS example, only the coefficients corresponding to acceleration measurements are plotted. It can be seen that for modeling of the longitudinal vibration, the absolute values of Coefficients 5-5 and 5-11 decrease, signifying substructural stiffness loss about the weak axis. For modeling of the transverse vibration, the negative correlation between acceleration from Nodes A3 and A8 increases after the structural change, suggesting that the substructural torsion behavior is affected more than the flexural behavior about the strong axis. The other two, Coefficients 8-4 and 8-12, did not report a statistically significant variation because the data insufficiency here has caused the modeling error to increase.

Table 6. Structural Change Characterization Results for Truss Bridge Member PP 22/PP 22

<table>
<thead>
<tr>
<th>Sensing locations associated with largest change in a damage indexa</th>
<th>Including translations at A11 and A12</th>
<th>Excluding translations at A11 and A12</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDIV from regression coefficients</td>
<td>Model 1</td>
<td>Model 2</td>
</tr>
<tr>
<td>NMS from regression coefficients</td>
<td>1</td>
<td>8</td>
</tr>
<tr>
<td>NDIV from RF1 and RF2</td>
<td>1/4-8</td>
<td>5</td>
</tr>
<tr>
<td>NMS from RF1 and RF2</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

The numbering of these locations coincides with the numbering of the accelerometers in Fig. 15.

Table 7. Most Probable Damage Sites from Voting of Different Suggested Locations for Truss Bridge Member PP 22/PP 22

<table>
<thead>
<tr>
<th>Damage locations ranked by the voting schemea</th>
<th>Including translations at A11 and A12</th>
<th>Excluding translations at A11 and A12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

Note: Bold numbers highlight locations associated with longitudinal vibration.

The numbering of these locations coincides with the numbering of the accelerometers in Fig. 15.
regression models, three damage indexes, and two CPA methods. Because only one intermediate translational response is available for modeling, the substructural beam model either disregards the substructural dynamic effects or takes it into consideration by introducing a lumped mass at the location of the intermediate measurement. Because the regression formulation can be carried out in both the time and frequency domains, a total of four regression models are obtained for data processing. The regression coefficients and regression residual variance ratios are selected as damage indexes because they can be efficiently obtained once the regression model is constructed. The two CPA techniques are selected based on the grounds of computational efficiency and performance reliability: only additive operations along data sequences are needed to locate the change point, and CPA methods are in general more reliable than statistical process control based on hypothesis testing.

The 24 linear regression/CPA based algorithms are fed into a voting-based classifier fuser for a final damage detection decision through merging their individual prediction results. This substructural methodology is applied to detect and localize damage in a laboratory 2-bay planar frame structure for validation of the in-plane beam model, and also to the structural change in vertical members of a simulated space truss and a steel truss bridge for validation of the space beam model. On the whole, the results from the applications here support the effectiveness of the proposed damage detection and localization scheme. It is thus observed that an array of simple classifiers, when properly combined, will have the ability to yield satisfactory results.

It is observed that different algorithms tend to yield different damage identification and localization results, and the performance of each algorithm varies by application. For example, the static model outperforms the dynamic model when the in-plane beam segment is shorter, whereas the reverse is true as the segment becomes more slender. This is because different models implemented in different settings have modeling errors to various degrees. Because beam elements in actual structures have continuous mass and those in FE modeling software can have more complex lumped mass patterns than those from substructural modeling, the beam elements proposed here are only approximations of the experimental and simulation beam components. Material and system nonlinearities in real structures could also contribute to the observed discrepancies. Other sources of modeling error include neglect of damping effects and, for space beam elements, the omission of torsional inertia in the model. These simplifications are done to avoid model overparameterization and to limit the regressors, where possible, to substructural boundary measurements such that the regression coefficients would reflect the structural condition between the interior node and boundary. The information synthesis procedure has helped to average out the model errors from distinct algorithms and achieve an accurate decision in most of the applications.

On a cross-comparison note, the coefficient estimates in the applications here generally demonstrate larger variations than those in Yao and Pakzad (2014), where regression was carried out for three consecutive stories within a shear frame structure. This is probably because substructural beam elements are less well-excited than mass-spring subsystems, yet can have more parameters to estimate if the elements are in 3D space. These observations evince the importance of the signal-to-noise ratio in structural damage detection applications.

Substructural damage detection techniques are often based on input-output data, and are relatively robust for excitation variations. The formulation of the substructural damage detection problem here prescribes that only input excitation acting directly on a substructure has an effect on its behavior. The excitations acting on the rest of the global structure influence the substructure through the boundary interaction forces, which are represented using the boundary measurements. In the derivations, it is assumed that excitation is random noise, which is a valid approximation for common ambient excitations on real structures. Therefore, as long as the substructural excitation remains largely chronologically uncorrelated, the proposed method yields stable performance.

Structural state identification results can be affected by different types of environmental factors: temperature, moisture, measurement noise, etc. If the measurement noise has statistical characteristics

![Fig. 17. Acceleration-related coefficients of the regression models for the longitudinal and transverse vibrations: (a) Model 3 with Location 5 as the regressand node; (b) Model 1 with Location 8 as the regressand node](figure)

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close to that of white noise, then its effect will be suppressed during the correlation/power spectrum—based regression process. Temperature and moisture variations, on the other hand, will alter the structural material constitutive relationships, thereby changing the structural stiffness. The proposed approach would not be able to distinguish such changes from those changes caused by structural damage; however, many other existing damage detection techniques, especially those based solely on structural vibration, also have similar limitations. To overcome this problem, these varying environmental parameters may need to be measured independently and incorporated into the modeling process.

The substructural concept can be applied to form elements of different geometry; the key issue is the identification of boundary reactions, external input, and environmental factors. Measurements that best represent these influences should be made with an adequate signal-to-noise ratio, and appropriate substructural models can then be constructed by expressing the substructural intermediate responses as functions of these measurements. Functions of the model parameters and residuals will then be used as substructural state indexes. This method could prove useful for monitoring of certain important parts of a large-scale structure, where sensors of various natures can be implemented to obtain high information density in that particular region. For other parts of the structure with less sensing density, existing macroscale system identification and monitoring schemes can still be applied.

Acknowledgments

Research funding is partially provided by the National Science Foundation through Grant No. CMMI-1351537, by the Hazard Mitigation and Structural Engineering program, and by a grant from the Commonwealth of Pennsylvania, Department of Community and Economic Development, through the Pennsylvania Infrastructure Technology Alliance (PITA).

References