

Modified Natural Excitation Technique for Stochastic Modal Identification

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Abstract: This paper presents an improvement to the eigensystem realization algorithm (ERA) with natural excitation technique (NExT), which is called the ERA-NExT-AVG method. The method uses a coded average of row vectors in each Markov parameter for evaluating modal properties of a structure. The modification is important because, for the existing stochastic system identification methods, the state-space model, obtained from output sensor data, is usually overparameterized resulting in large systems. Solving such a problem can be computationally very intensive especially in the applications when using the computational capabilities of embedded sensor networks. As a way to improve the efficiency of the ERA-NExT method, the proposed method focuses on the number of components in a single Markov parameter, which can theoretically be minimized down to the number of structural modes. Applying the coded average column vectors as Markov parameters to the ERA, the computational cost of the algorithm is significantly reduced, whereas the accuracy of the estimates is maintained or improved. Numerical simulations are performed for a shear frame model subjected to Gaussian white noise ground excitation. The efficiency of the proposed method is evaluated by comparing the accuracy and computational cost of the estimated modal parameters using the proposed method, with several other stochastic modal identification methods including the ERA-observer Kalman filter identification, ERA-NExT, and autoregressive models. The performance of the method is then evaluated by applying it to ambient vibration data from the Golden Gate bridge, collected using a dense wireless sensor network, and its vertical and torsional modes are successfully and accurately identified. DOI: [10.1061/\(ASCE\)ST.1943-541X.0000559](https://doi.org/10.1061/(ASCE)ST.1943-541X.0000559). © 2013 American Society of Civil Engineers.

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Introduction

In the last two decades, significant efforts in the research community have focused on development of methodologies and applications to construct numeric models in the control area. Implementing system identification methods for the dynamic response of civil infrastructure, structural systems, or machinery has facilitated the creation of increasingly accurate and updated mathematical models (Ljung 2008). The role of system identification algorithms to estimate precise values for these structural properties has become more significant for damage detection applications, because damage in a structural system affects the structural properties such as stiffness and modal properties (Li and Zhang 2006).

The identification procedure starts with modeling the input/output relationship, which can be established with a time domain finite-difference model. This relationship is known as the autoregressive model with exogenous input (ARX), where the coefficients represent the dynamic characteristics of the structural system. To estimate these parameters, the eigensystem realization algorithm (ERA) was proposed by Juang and Pappa (1985), which has served

as one of the most widely used system identification methods. In its original formulation, the ERA method uses the impulse response of a dynamic system (i.e., the Markov parameters), which is difficult to estimate for existing structures (Phan et al. 1993; Arici and Mosalam 2005a, b). Several studies have focused on overcoming the limitations of this algorithm in the last few decades (James et al. 1993; Juang 1994). The observer/Kalman filter is used to remedy the lack of initial conditions, extend the applicability of the ERA to excitations other than impulse, and create unique system Markov parameters (Juang 1994). James et al. (1993) proposed the ERA-natural excitation technique (NExT) using auto- and cross-correlation functions of output data to extend the application of the algorithm further to stochastic (output-only) systems. This method has been adopted in several structural identification studies using sensor data (Farrar and James 1997; Caicedo et al. 2004; Brownjohn 2003). The operational modal analysis (OMA) is proposed by Mohanty and Rixen (2004, 2006), where the ERA-NExT is modified to be applicable to pure harmonic input and white noise input. Although this approach is useful for mechanical systems, which include rotating machinery, it is not very applicable for civil infrastructure where input data are hardly generalized to a single harmonic function.

Each system identification method requires a certain type of data such as the impulse response, the input/output response, or only the output response. Accordingly, the model order to obtain the correct modal properties with comparable efficiency and accuracy varies depending on the system identification method. As a result, understanding the advantages and disadvantages of each method and determining the most appropriate method to implement in different applications are important. For instance, Lew et al. (1993), in a comprehensive study compared the ERA (Juang and Pappa 1985), ERA using data correlation (ERA/DC) (Juang et al. 1988), Q-Markov cover theory (Anderson and Skelton 1988), and an algorithm proposed

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by Moonen et al. (1989). It was concluded that the ERA/DC can be the best identification method among the four for input-output data. Petsounis and Fassois (2001) compared the identified modal parameters using a stochastic method based on the autoregressive moving average (ARMA) or ARX model. However, the necessity for input and output data for the ERA decreases its practical applicability for system identification of some existing structures, where stochastic methods are preferred. Peeters et al. (1998) assessed stochastic system identification methods including the peak picking (PP), polyreference least-square complex exponential (LSCE), and stochastic subspace identification (SSI) methods (Van Overschee and De Moor 1991). The results show that the quality of the identified mode shapes from SSI has been better than the other methods. These comparison studies have only focused on the accuracy of the results of identification methods and do not provide extensive information on computational cost for each method. The performance measures should be combined with the efficiency of each algorithm to present a comprehensive comparison.

In this paper, a modified ERA-NExT algorithm, known as the ERA-NExT-AVG, is proposed, where the size of Markov parameters is systematically reduced to optimize computational time. To investigate the efficiency of the proposed method, the following two aspects of the algorithm are studied: (1) the accuracy of identified modal parameters and (2) the computational cost. At first, the formulation of the proposed ERA-NExT-AVG algorithm is introduced. Then, the framework for benchmark tests to estimate the performance of system identification algorithms is presented. The third section presents the results of system identification based on the proposed method for the numerically simulated data and the benchmark study to compare various system identification methods including the ERA-observer Kalman filter identification (OKID), ERA-NExT, and Auto Regressive (AR) methods (Pakzad and Fenves 2009). The last section shows the application of the proposed method to identify the vertical and torsional modes of Golden Gate bridge (GGB), using ambient vibration data from a dense wireless sensor network.

Eigensystem Realization Algorithm for Modal Identification

The ERA method uses a system's impulse response to derive controllability and observability without considering external force terms in its formulation (Juang and Pappa 1985). To understand the fundamental concept of the ERA method, the discretized state-space equation, which expresses the equation of motion for a linear time invariant (LTI) system in a first-order difference equation, is considered as

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned} \quad (1)$$

In Eq. (1), $x(k) \in \mathfrak{R}^{2N}$ = state vector, N = number of degrees of freedom (DOF), $u(k) \in \mathfrak{R}^n$ = input vector at n locations of input to the structure, and $y(k) \in \mathfrak{R}^m$ = output vector at m locations of response on the structure (nodes). The coefficients (A , B , C , D) are called the discretized state, input, output, and feed-through matrices, respectively.

The Hankel matrix $H(k-1)$ is formed by rearranging impulse response $y(k)$, which produces the system Markov parameters to identify the N rank of the state matrix

$$H(k-1) = \begin{bmatrix} y(k) & y(k+1) & \cdots & y(k+q-1) \\ y(k+1) & y(k+2) & \cdots & y(k+q) \\ \vdots & \vdots & \ddots & \vdots \\ y(k+p-1) & y(k+p) & \cdots & y(k+p+q-2) \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} CA^{k-1}B & CA^k B & \cdots & CA^{k+q}B \\ CA^k B & CA^{k+1}B & \cdots & CA^{k+q}B \\ \vdots & \vdots & \ddots & \vdots \\ CA^{k+p-2}B & CA^{k+p-1}B & \cdots & CA^{k+p+q-3}B \end{bmatrix} \\ &= P_p A^{k-1} Q_q (k \geq 1) \end{aligned} \quad (2)$$

where two block matrices are defined as the extended observability matrix $P_p = [C^T (CA)^T \cdots (CA^{p-1})^T]^T$ and the extended controllability matrix $Q_q = [B AB \cdots A^{q-1}B]$.

Every two successive Hankel matrices provide a simple approach to obtain the state matrix because these are related implicitly to a system. For instance, the Hankel matrices, $H(0)$, $H(1) \in \mathfrak{R}^{mp \times nq}$ can be written as

$$\begin{aligned} H(0) &= P_p Q_q \\ H(1) &= P_p A Q_q \end{aligned} \quad (3)$$

The singular value decomposition (SVD) is applied to the Hankel matrix, $H(0)$, to derive a possible set for block matrices P_p and Q_q as

$$H(0) = R \Sigma S^T = [R_1 \quad R_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} S_1^T \\ S_2^T \end{bmatrix} \quad (4)$$

where $P_p = R_1 \Sigma_1^{1/2}$, $Q_q = \Sigma_1^{1/2} S_1^T$, $\Sigma_1 = \text{diag}[\sigma_1 \quad \sigma_2 \quad \dots \quad \sigma_{N_s}]$, and N_s = number of modes. In the ideal case with no measurement noise or modeling error, the system has N_s nonzero singular values, and the rest of the singular values are zero. In practice, however, a threshold is needed to distinguish and discard the insignificant singular values, which often results in a system that is much larger than the physical system. In the efficiency comparison, however, N_s is assumed to be equal to nq because (1) there is no automatic measure to establish this threshold and (2) this model reduction technique can be applied to other modal identification methods as well, so it is reasonable to compare all of the methods for their largest possible set of modes. The transition state and output matrices ($\hat{A} \in \mathfrak{R}^{N_s \times N_s}$ and $\hat{C} \in \mathfrak{R}^{m \times N_s}$) are calculated as

$$\begin{aligned} \hat{A} &= \Sigma_1^{-1/2} R_1^T H(1) S_1 \Sigma_1^{-1/2} \\ \hat{C} &= [I_m \quad 0 \quad 0 \quad \dots \quad 0] P_p \end{aligned} \quad (5)$$

In Eq. (5), $I_m \in \mathfrak{R}^{m \times m}$ = identity matrix, and $0 \in \mathfrak{R}^{m \times m}$ = zero matrix. The eigenvalue decomposition for the transition state matrix \hat{A} is used to obtain the poles, which is $\Lambda = \text{diag}(\lambda_i)$, $i = 1, 2, \dots, N_s$. Natural frequencies and damping ratios of the system are given by

$$\begin{aligned} \mu_i &= \frac{\log(\lambda_i)}{\Delta t}, \quad \mu_i, \mu_i^* = -\omega_i \zeta_i \pm j\omega_i \sqrt{1 - \zeta_i^2} \\ \omega_i &= \sqrt{\mu_i \cdot \mu_i^*} \quad (* \text{ conjugate form}) \\ \zeta_i &= -\frac{\text{Real}(\mu_i)}{\omega_i} \end{aligned} \quad (6)$$

In Eq. (6), the subscript i is varied from 1 to N_s and Δt = sampling period. See Juang and Pappa (1985), Juang et al. (1988), and Chen et al. (1992) for more information about the ERA.

Because of the measurement noise, systematic error, non-linearity, and computational error, nearly similar rank is observed between the Hankel matrix $H(0)$ and transition state matrix \hat{A} . Therefore, the system realization produces noisy sequences with poles that are not from the real system, leading to identification of

spurious computational and noise modes. A stabilization diagram (Heylen et al. 1995) is usually used to identify the real structural modal parameters from the overparameterized state matrix.

Natural Excitation Technique for Stochastic Models

The underlying theory for the NExT is that the auto- and cross-correlation functions of output data for a system subjected to Gaussian white noise are similar to the impulse response (James et al. 1993). The correlation function $R_{ij}(T)$ between response at nodes i and j in terms of time lag T is expressed as

$$R_{ij}(T) = \sum_{r=1}^N \left[\frac{\phi_i^r Q_j^r}{m^r \omega_d^r} \exp(-\zeta^r \omega_n^r T) \sin(\omega_d^r T + \theta_r) \right] \quad (7)$$

In Eq. (7), the superscript r = particular mode from a total of N modes, ϕ_i^r = i th ordinate of the r th mode shape, m^r = r th modal mass, Q_j^r = constant associated with response at node j , ζ^r and ω_n^r = r th mode damping ratio and natural frequency, respectively, $\omega_d^r = \omega_n^r \sqrt{1 - (\zeta^r)^2}$ = r th mode damped natural frequency, and θ_r = phase angle associated with the r th modal response. The total number of correlation functions corresponding to a particular time lag is m^2 , and the Markov parameter $R(T)$, representing impulse response, is composed of those correlation functions as

$$R(T) = \begin{bmatrix} R_{11}(T) & R_{12}(T) & \cdots & R_{1m}(T) \\ R_{21}(T) & R_{22}(T) & \cdots & R_{2m}(T) \\ \vdots & \vdots & \ddots & \vdots \\ R_{m1}(T) & R_{m2}(T) & \cdots & R_{mm}(T) \end{bmatrix} \quad (8)$$

Similar to the ERA method with impulse response, the Hankel matrices $H(0)$, $H(1) \in \mathfrak{R}^{mp \times nq}$ are obtained by substituting $k = 0$ and 1 in Eq. (9)

$$H(k) = \begin{bmatrix} \bar{R}(k) & \bar{R}(k+1) & \cdots & \bar{R}(k+q-1) \\ \bar{R}(k+1) & \bar{R}(k+2) & \cdots & \bar{R}(k+q) \\ \vdots & \vdots & \ddots & \vdots \\ \bar{R}(k+p-1) & \bar{R}(k+p) & \cdots & \bar{R}(k+p+q-2) \end{bmatrix} \quad (9)$$

In Eq. (9), $\bar{R}(k) = R(k\Delta t)$, and its components are $\bar{R}_{ij}(k) = E[y_i(n+k)y_j(n)]$. The other steps of the identification process, estimating the equivalent state space model such as (\hat{A}, \hat{C}) and determining modal parameters using eigenvalue decomposition, are similar to the ERA method with impulse response.

Modified Natural Excitation Technique Using Average Markov Parameters

The main idea of the ERA-NExT-AVG method is that the size of each Markov parameter in the existing NExT method can be reduced to increase the efficiency of the algorithm and avoid redundant data. The size of each Markov parameter is $(m \times m)$ with autocorrelation functions for the diagonal elements and cross-correlation functions for the off-diagonal elements. A single output data set y_i is used $2m - 1$ times to create a Markov parameter, in which all elements of each i th row and column ($R_{i1}, R_{i2}, \dots, R_{im}$ and $R_{1i}, R_{2i}, \dots, R_{mi}$) are calculated using the same response at sensor location i . Considering that the estimated state matrix is full rank in the presence of the measurement and modeling noise, there are excessive data in

a Markov parameter estimated by the NExT, whereas only N rank is required for Hankel matrix to extract an equivalent state space model. These excessive data in Markov parameters add a heavy computational burden and increase the computational noise in the eigenvalue decomposition. To identify the mode shapes, the optimum number of independent elements in a Markov parameter is m , which is regarded as a maximum number of modes to be identified. Alternatively, a single reference-based correlation can be considered as a Markov parameter; however, using a single reference may not capture all of the structural modes when a certain structural mode has a modal node at the reference location. The proposed ERA-NExT-AVG method addresses this problem by considering all of the Markov parameters.

To overcome this problem, the modal properties of a system are identified when the values in a row of the cross-covariance matrices are averaged based on the contribution of each mode as

$$\begin{aligned} \hat{R}_i(T) &= \frac{1}{m} \sum_{j=1}^m \alpha_j R_{ij}(T) \\ &= \sum_{r=1}^N \left[\left(\frac{1}{m} \sum_{j=1}^m \alpha_j Q_j^r \right) \frac{\phi_i^r}{m^r \omega_d^r} \exp(-\zeta^r \omega_n^r T) \sin(\omega_d^r T + \theta_r) \right] \end{aligned} \quad (10)$$

In Eq. (10), the subscript i is varied from 1 to m , and α_j = coding coefficient associated with the j th column of the Markov parameter $R(T)$. The reason for applying this coding coefficient is to avoid eliminating modes by adding all components together. For instance, antisymmetric modes for a symmetric bridge system subjected to white noise excitation are canceled out by adding the responses together. The coding coefficients prevent this from happening by assigning different weights to different response signals, thus avoiding the antisymmetric sums from becoming zero and then compensating for this by decoding the modal ordinates. The coding coefficients can be selected based on an a priori and approximate knowledge of the mode shapes of the system to improve the performance of the method. For example, in the case of a long span bridge with distributed mass and elasticity, they could be considered to have a harmonic shape. The number of the mode shapes considered here is assumed to be equal to the number of sensing nodes on the structure. The coding coefficients are obtained by post-multiplying the pseudo-inverse of the assumed mode shapes by a weighting vector as

$$\{\alpha\} = \{\iota\} \cdot \begin{bmatrix} \tilde{\phi}_{11} & \tilde{\phi}_{12} & \cdots & \tilde{\phi}_{1m} \\ \tilde{\phi}_{21} & \tilde{\phi}_{22} & \cdots & \tilde{\phi}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\phi}_{m1} & \tilde{\phi}_{m2} & \cdots & \tilde{\phi}_{mm} \end{bmatrix}^\dagger = \{\iota\} \cdot [\tilde{\Phi}]^\dagger \quad (11)$$

In Eq. (11), $\tilde{\phi}_{ij}$ = modal ordinate at location i for the j th mode shape, $\{\iota\}$ = weighting vector, and the $[\tilde{\Phi}]^\dagger$ = pseudo-inverse of an assumed modal matrix.

The average coefficient $\hat{R}_i(T)$, is also a function of time lag T with decaying harmonic functions, and the average value of $\alpha_j Q_j^r$ is a constant. Finally, an average correlation function that is also a Markov parameter can be expressed with only m terms as

$$\hat{R}(T) = \left[\hat{R}_1(T) \quad \hat{R}_2(T) \quad \dots \quad \hat{R}_m(T) \right]^T \quad (12)$$

Physically, each parameter defined previously represents the correlation of one node with the weighted average of every other node in the system as a function of the time lag

$$\hat{R}(T) = \frac{1}{m} \sum_{j=1}^m E \left[y_i(t+T) \cdot \alpha_j y_j(t) \right] \\ = E \left\{ y_i(t+T) \cdot \left[\frac{1}{m} \sum_{j=1}^m \alpha_j y_j(t) \right] \right\} \quad (13)$$

The contribution of measurement noise and model error in the response at node j is difficult to evaluate, especially for field tests. The intensity of noise, however, can be reduced by taking the average of the cross-correlation of the response of a node with every other node, which involves addition and multiplication operations. The number of Markov parameters in a column of a Hankel matrix is chosen as half of the product of model order and the number of components in the coded average Markov parameters $[\hat{R}(T)]$, to avoid biased information on its column elements.

The SVD function creates an almost fully ranked state matrix. Therefore, the size of the estimated state matrix needs to be minimized to avoid identifying undesired spurious modes. The proposed ERA-NExT-AVG method provides an effective solution to minimize the size of Markov parameters. Furthermore, it is expected that a reduced number of elements in each Markov parameter reduces computational time, especially for the calculation of SVD, eigenvalue analysis, and other matrix operations.

Stabilization Diagram

The dimension of the state matrix in the ERA affects the number of identified modes. As a result, this number usually exceeds the number of structural modes. Heylen et al. (1995) proposed using stabilization diagrams to eliminate spurious modes and choose the minimum model order by examining the cohesion of identified modal parameters. A threshold is set to monitor the convergence of the identified frequencies as the model order is increased. The frequencies of the true structural modes tend to converge as the model order changes, whereas for the spurious computational modes, convergence is erratic. Similar measures are applied to damping ratios and the modal assurance criterion (MAC) introduced by Allemang and Brown (1982) to identify converging structural modes. The MAC value between mode shapes $\tilde{\varphi}_k$ and $\tilde{\varphi}_k$ is defined as

$$MAC_k = \frac{(\tilde{\varphi}_k^* \tilde{\varphi}_k)^2}{(\tilde{\varphi}_k^* \tilde{\varphi}_k)(\tilde{\varphi}_k^* \tilde{\varphi}_k)}, \quad k = 1, \dots, N_s \quad (14)$$

where the superscript $*$ denotes the conjugate transpose. In each case, the three thresholds for frequency, damping ratio, and mode shape are applied to the identified modal parameters as the model order increases, and once convergence is achieved, a mode is marked as an identified structural mode.

Efficiency Evaluation

The performance of the proposed method is assessed by taking into account the accuracy of the identified results and efficiency of the algorithm for a simulated example. Accuracy and efficiency are conflicting objectives with respect to the model order; an increase of model order results in improvement of the quality of identified results but an increase in the computational cost. With this

background, this section provides the comparison bases by quantifying the accuracy and efficiency measures for each modal identification method.

Accuracy Tests

For the simulated example presented in the next section, the accuracy of the identification method is evaluated by studying the relative estimation error for the identified modal parameters compared with the exact values. For the frequencies and damping ratios, the normalized relative errors are used, where the exact value of the parameter is used as the normalization factor. The errors (ε) in natural frequency, damping ratio, and MAC value between identified and exact modes are defined as

$$\varepsilon_{\omega_n} = \left| 1 - \frac{(\omega_n)_{id}}{(\omega_n)_{exact}} \right| \\ \varepsilon_{\zeta} = \left| 1 - \frac{\zeta_{id}}{\zeta_{exact}} \right| \quad (15) \\ \varepsilon_{MAC} = 1 - \frac{(\varphi_{id}^* \varphi_{exact})^2}{(\varphi_{id}^* \varphi_{id})(\varphi_{exact}^* \varphi_{exact})}$$

In Eq. (15), the subscripts id and $exact$ = identified and exact modal parameters, respectively. The estimation errors defined by this equation are used to evaluate the accuracy of the proposed method compared with the other stochastic identification methods.

Computational Costs

In general, system identification methods can be computationally intensive algorithms, and the computational cost becomes more significant as the size of the system increases. It is shown in the recent implementations of sensor networks that more sensor data with higher sampling rates are required to achieve more accurate identification results (Pakzad and Fenves 2009). This will cause a heavy computational task for a desired accuracy in a monitoring systems; doubling the model order results in quadrupling the size of the state matrix and a significant increase in the computational time. In addition, the recent applications of WSNs in vibration monitoring have the potential of creating a spatially dense data grid with limited embedded computational capacity. Reducing the

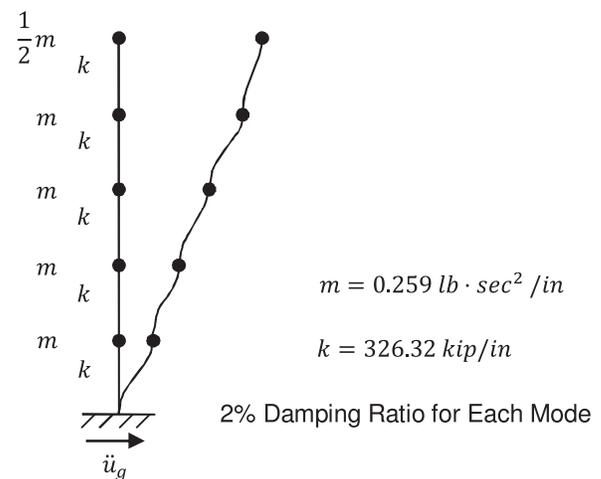


Fig. 1. Simulated 5 DOF shear frame structure

computational burden of data analysis is essential for viability of such networks.

The efficiency of the proposed method is assessed by estimating the computational cost of the algorithm and comparing it to other system identification methods. Two different measures are used for comparison: (1) the total number of operations for each model order and (2) the actual computational time (CPU time). Although estimating the computational time is an easier task, it depends on the system hardware, software, and the programming code. However, the total number of operations is a more accurate measure and only depends on the size of the problem in hand. The uncertainty in estimating the number of operations comes from the fact that the eigenvalue decomposition and SVD processes include iterations and their convergences depend on the characteristics of a given matrix. The numbers of operations for eigenvalue decomposition and SVD procedures are approximated based on the Linear Algebra PACKAge (LAPACK) benchmark (Anderson et al. 1999), which provides an estimate for the number of

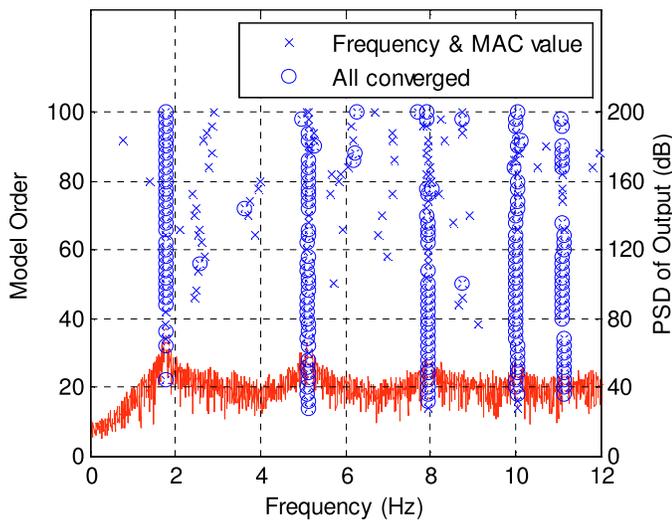


Fig. 2. Stabilization diagram using ERA-NExT-AVG method for the simulated model subjected to white noise excitation

operations in terms of floating point operation (flop). Therefore, the two approaches (number of operations and CPU time) complement each other in estimating the computational cost for a specific algorithm. A detailed description of this comparison for different modal identification methods is presented in the following sections.

Simulation and Comparison

Modal Identification Using the Modified Eigensystem Realization Algorithm-Natural Excitation Technique Method

A 5-story shear frame structure is chosen as the simulated testbed to estimate the accuracy and computational cost of the identification. Classical damping is applied by assuming 2% damping ratio for each mode (Fig. 1). The structural parameters including mass, damping, and stiffness matrices are specified accordingly. To generate the numerical response of the system, Newmark's linear acceleration method is used (Newmark 1959). After simulating the acceleration response of the system, white noise with a SD equal to 5% RMS of the response is added to each output signal to simulate measurement noise.

The harmonic functions are assumed as the starting for mode shapes of a shear building model with a fixed support at base. The coding coefficients $\{\alpha_{ex}\}$ for the given structure are determined as

$$\{\alpha_{ex}\} = \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}^T \cdot \begin{bmatrix} 0.10 & 0.35 & 0.65 & 0.90 & 1.00 \\ 0.35 & 0.90 & 0.81 & -0.31 & -1.00 \\ 0.65 & 0.81 & -0.81 & -0.31 & 1.00 \\ 0.90 & -0.31 & -0.31 & 0.81 & -1.00 \\ 1.00 & -1.00 & 1.00 & -1.00 & 1.00 \end{bmatrix}^{\dagger}$$

$$= \begin{Bmatrix} 1.30 \\ 0.52 \\ 0.40 \\ 0.32 \\ 0.15 \end{Bmatrix}^T \quad (16)$$

(16)

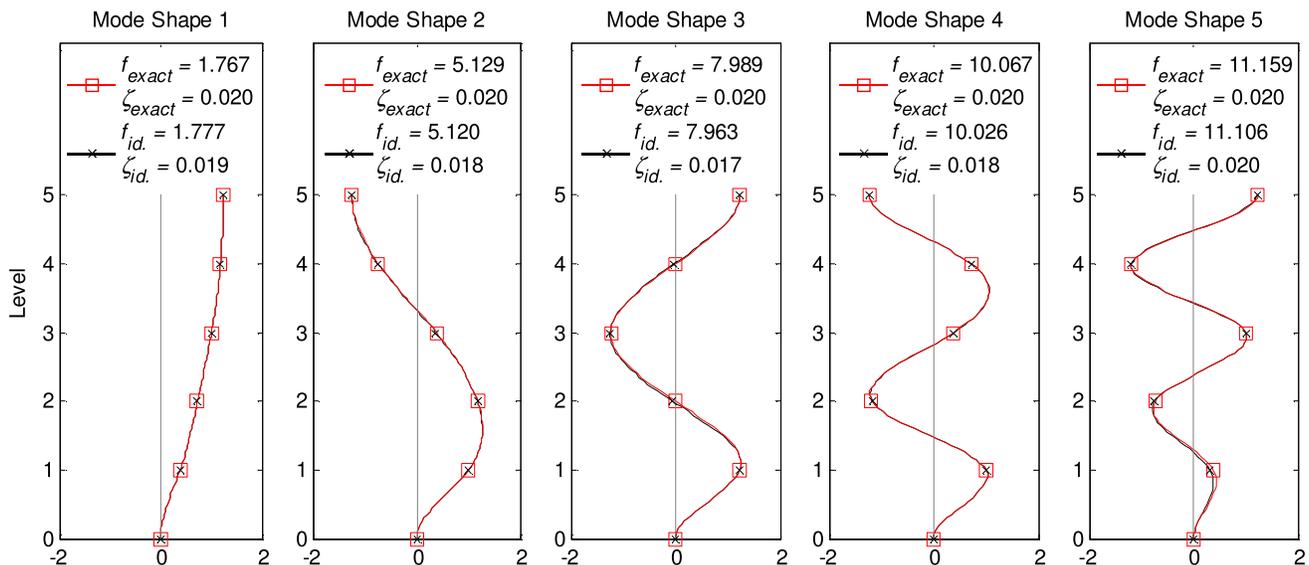


Fig. 3. Comparison of identified modal parameters using ERA-NExT-AVG method with the true values for model order 50

Table 1. Number of Operations for the ERA-NExT-AVG Method

Description	Dimension	Number of operations (flops)
SVD for $H(0)$	$d \times c$	$[6.67d^3]$
$A = \Sigma_n^{-1/2} \cdot R_n^T \cdot H(1) \cdot S_n \cdot \Sigma_n^{-1/2}$	$c \times c$	$[2cd(c+d)] + \{c[2c(c+d) - (3c+d)]\} + [8c]$ <small>multiplication summation root</small>
$C = I_p \cdot R_n \cdot \Sigma_n^{1/2}$	$m \times c$	$[mc(c+d)] + [mc(c+d-2)] + [8c]$ <small>multiplication summation root</small>
Eigenvalue and eigenvector of A	$c \times c$	$[26.33c^3]$ <small>eigenvalue estimation</small>
Modal parameters		$[mc^2 + 3c] + [mc(c-1) + c] + [8c] + [20c]$ <small>multiplication summation root log</small>

Note: $c = \langle m/2 \rangle \cdot p$; $d = mp$.

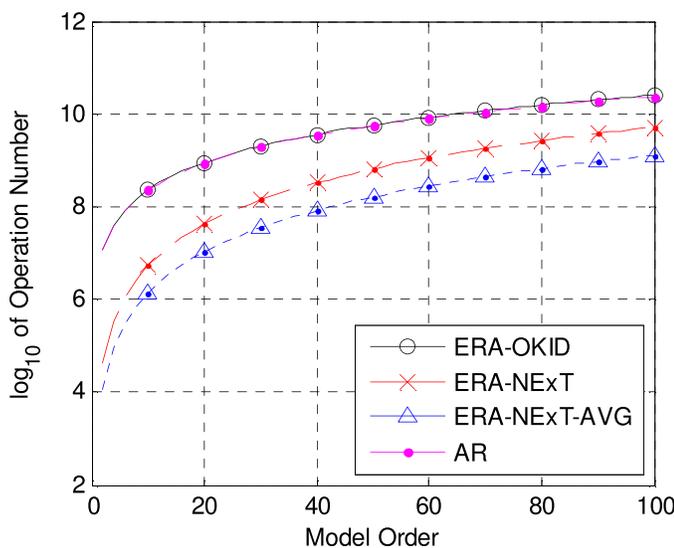


Fig. 4. Comparison of the number of operations for output-only system identification methods: ERAOKID, ERA-NExT, ERA-NExT-AVG, and AR model

The 5×5 matrix in Eq. (16) is the modal matrix of the structure. The weighting vector is assumed to be equal to unity, assigning equal weight to all sensing nodes considered for the coding coefficients. The correlation functions are then multiplied by the coding coefficients $\{\alpha_{ex}\}$. By applying different weightings for correlation functions, the proposed method avoids canceling modes and improves on the quality of identification results.

The results of the modal identification using the ERA-NExT-AVG are shown in the form of its stabilization diagram (Fig. 2). In the stabilization diagram, the \times sign indicates that the identified damping ratio and the frequency have converged for a particular mode. The \circ sign means the mode has converged for frequency, damping ratio, and MAC value. The average of the power spectral density of output responses is also included in the stabilization diagrams for reference (in dB). To identify true modes, the convergence thresholds for frequencies, damping ratios, and MAC values are set to 0.05, 0.1, and 0.95, respectively.

The stabilization diagram shows that the structural modes can be consistently extracted from the data. On the basis of the stabilization

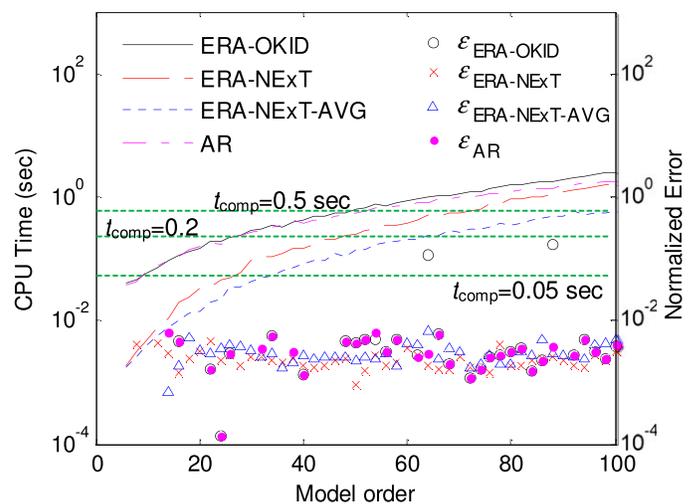


Fig. 5. Comparison of the CPU time for output-only system identification methods: ERA-OKID, ERA-NExT, ERA-NExT-AVG, and AR model; in each case, the corresponding normalized estimation error for frequency and MAC value are also included

Table 2. Efficiency of Each System Identification Method Corresponding to Certain Computation Times

CPU time (s)	Identified model order, normalized error			
	ERA-OKID	ERA-NExT	ERA-NExT-AVG	AR
0.05	(8, fail)	(26, 0.27%)	(32, 0.26%)	(10, fail)
0.2	(22, 0.16%)	(42, 0.17%)	(56, 0.29%)	(22, 0.16%)
0.5	(40, 0.13%)	(62, 0.27%)	(90, 0.27%)	(40, 0.13%)

diagram, a model order of 32 is sufficient for the first mode of the simulated structure; similarly, the minimum model order for other modes can be determined. The identified mode shapes closely match the exact model as shown in Fig. 3, which illustrates the mode shapes of a numeric model and the identified system. The frequencies and damping ratios for the identified modes and the exact values are also indicated in this figure. The identified frequencies are very close to the exact values, as their relative error is less than 1%. The identification errors for the damping ratios are as high as 23%, which is

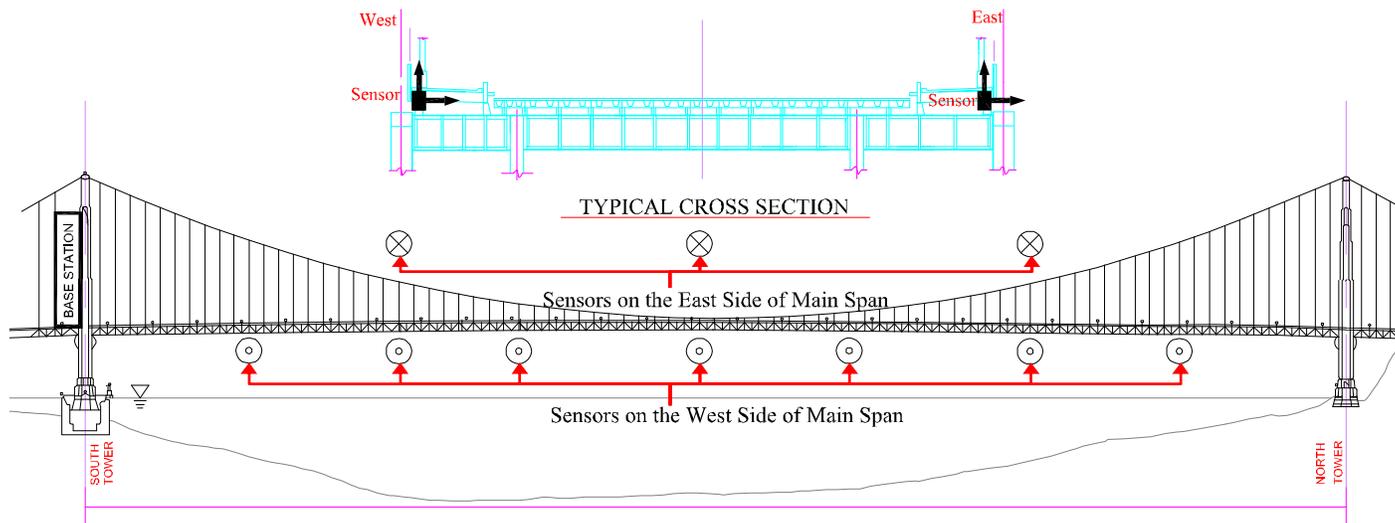


Fig. 6. Locations of 10 wireless accelerometer nodes on the main span of GGB

consistent with the performance of other stochastic modal identification methods.

Efficiency Comparison

Table 1 presents the number of operations in each task for the ERA-NExT-AVG method as a function of the number of outputs (m) and the model order (p). The major contributors to the computational cost are the eigenvalue decomposition and SVD, which comprise nearly 80% of the total number of operations. Minka (2010) provides further information about the flops for each operation. To compare the performance of several system identification methods, a similar approach is applied to estimate the number of operations for the ERA-OKID, ERA-NExT, and AR method. The number of operations shown in logarithmic scale versus model order demonstrates that the ERA-NExT-AVG is the least computationally intensive method among the ones investigated in this study as shown in Fig. 4. Because the operations for eigenvalue decomposition and SVD are each a cubic functions of the model order, the modal identification of the simulated 5 DOF system in this numerical example with the ERA-NExT-AVG requires less than 1% of operations compared with the ERA-NExT.

The CPU time is measured using the built-in function in *Matlab* (MathWorks 2009) for each identification method applied to this 5 DOF numeric example, and the results are plotted in Fig. 5 in logarithmic scale versus the model order. The normalized error (ϵ_{norm}) between the exact and estimated modal parameters are also plotted in the same graph and considered as a measure of accuracy. This measure is defined as the average value of errors in identified frequencies (ϵ_{ω_n}) and MAC values (ϵ_{MAC}) for each model order as

$$\epsilon_{\text{norm}} = \frac{\sum_{i=1}^{N_{id}} (\epsilon_{\omega_n, i} + \epsilon_{\text{MAC}, i})}{2N_{id}} \quad (17)$$

In Eq. (17), N_{id} is the number of identified modal parameters for a particular model order. For this simulated example, the ERA-NExT-AVG method takes the least CPU time, which means this method is the most efficient among the methods considered. The overall trend of the CPU is very similar to the plot for the number of operations.

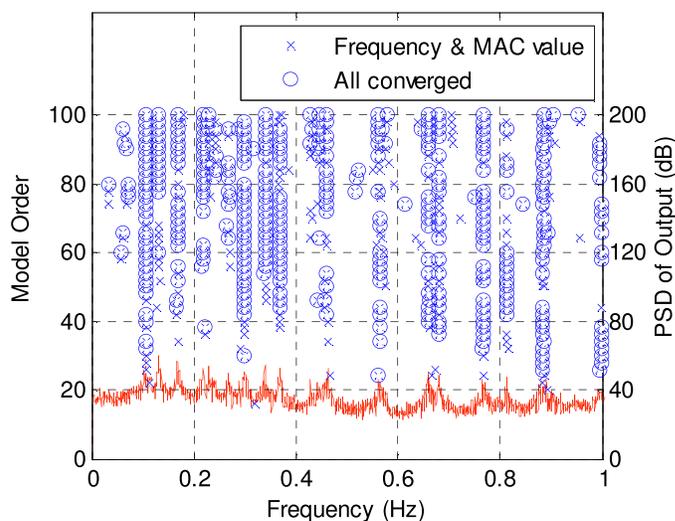


Fig. 7. Stabilization diagram using ERA-NExT-AVG method for ambient acceleration data from GGB

Table 2 shows the highest model order that can be achieved for the 5 DOF simulated system for specific CPU times and its corresponding normalized estimation errors. The results indicate that the errors in frequencies and MAC values are less than 0.3% for the methods that identify the structural modes successfully. Although for a specific CPU time the proposed method shows similar results compared with the ERA-NExT in terms of estimation accuracy, its efficiency improves over the ERA-NExT by allowing for a higher model order. Considering that the field tests normally require many sensors, the efficiency of identification algorithm becomes a significant factor for a robust method. The accuracy of the proposed method in estimating modal parameters is consistent with the other stochastic system identification methods that were considered.

Verification of the Method with Field Experiments

To evaluate the performance of the proposed method using field data, ambient vibrations of the GGB obtained by a wireless sensor

network are used. The acceleration data were measured using 65 wireless sensing nodes during a 3-month deployment period (Pakzad et al. 2008; Pakzad and Fenves 2009; Pakzad 2010). For this application, vertical acceleration data from 10 sensor nodes on the main span of the bridge are considered to investigate the vertical and torsional vibration modes with frequency up to 1 Hz. The location of the sensor nodes is shown in Fig. 6, where seven nodes are mounted on the west side of the bridge and the other three sensors are mounted on the east side. The sensors on the east side serve to distinguish between the vertical and torsional modes. Each node collected 80,000 samples of ambient vibrations at 50-Hz sampling frequency over 26.5 min.

In this section, the identified modes using the ERA-NExT-AVG method are presented and compared with the modes identified using

Table 3. Identified Frequencies and Damping Ratios for Eight Vertical and Six Torsional Modes from the Acceleration Response of the Main Span of GGB

Vertical mode			Torsional mode		
Mode	Frequency (Hz)	Damping ratio (%)	Mode	Frequency (Hz)	Damping ratio (%)
1A	0.1062	0.48	1A	0.2298	0.70
2S	0.1324	0.93	2S	0.3399	0.26
3S	0.1683	0.45	3A	0.4442	0.34
4A	0.2173	1.20	4S	0.5622	0.40
5S	0.2693	1.37	5A	0.6799	0.12
6S	0.3000	0.61	6S	0.8131	0.37
7A	0.3686	0.35			
8S	0.4608	0.60			

Note: A = antisymmetric mode; S = symmetric mode.

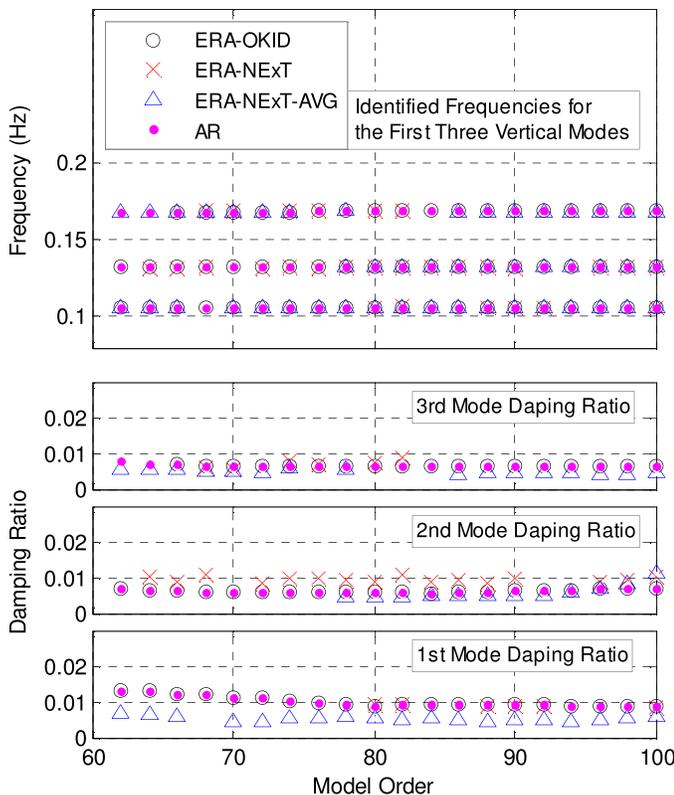


Fig. 8. Convergence of the identified frequency and damping ratio for the first three vertical modes of GGB as a function of the model order

the ERA-OKID, ERA-NExT, and AR methods. Overall, eight vertical and six torsional modes with a frequency < 1 Hz were identified. The convergence thresholds for stabilization diagram, shown in Fig. 7, were set to 0.1, 0.3, and 0.9 for frequency, damping ratio, and MAC value, respectively. In the stabilization diagram, identified modes present a very good match with the peaks of the power spectral density function. The identified modal frequencies and damping ratios are summarized in Table 3. The largest estimation errors occur for the damping ratios, which is consistent with the findings of Sun and Lu (1995) about the sensitivity of the damping estimates to the measurement noise. Figs. 8 and 9 show the changes in frequencies and damping ratios when the model order increases from 60 to 100 for the first three vertical and torsional modes. These figures show that a consistent and stable identification is achieved through the ERA-NExT-AVG method. The identification results using the ERA-OKID, ERA-NExT, and AR methods are also included in these figures, which show similar estimation behavior.

Fig. 10 shows a comparison of the first three vertical modes obtained from the ERA-NExT-AVG method, as well as ERA-OKID, ERA-NExT, and AR methods. The identified modes are generally consistent for all four methods. However, the CPU time for the ERA-NExT-AVG is consistently smaller than the other three methods (approximately 20% of the ERA-OKID, 27% of the ERA-NExT, and 50% of the AR method). Fig. 11 shows similar results for the first three torsional modes. The CPU time comparison for other stochastic identification methods supports the hypothesis that the proposed ERA-NExT-AVG method is more efficient in modal identification compared with the existing methods and can reduce the number of computation flops for eigenvalue estimation procedures.

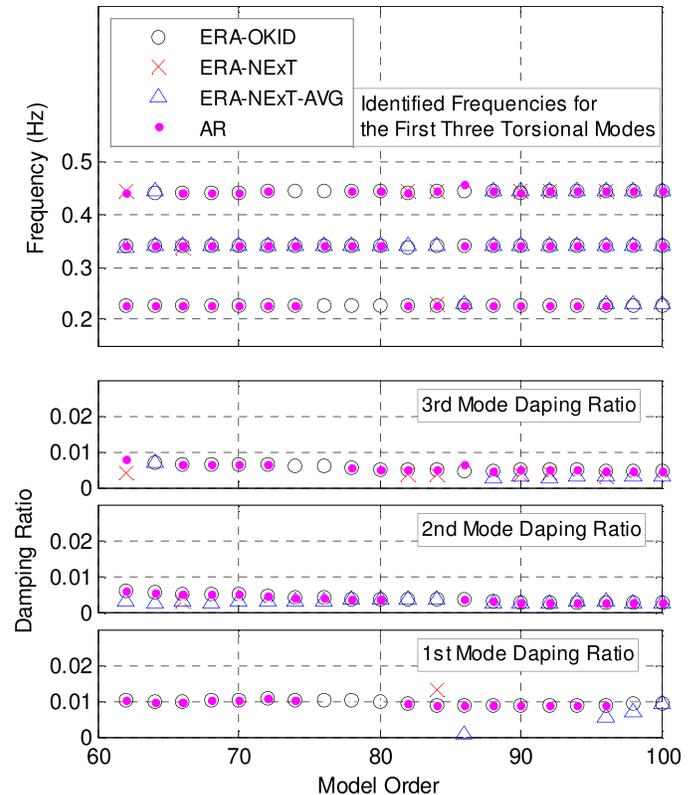


Fig. 9. Convergence of the identified frequency and damping ratio for the first three torsional modes of GGB as a function of the model order

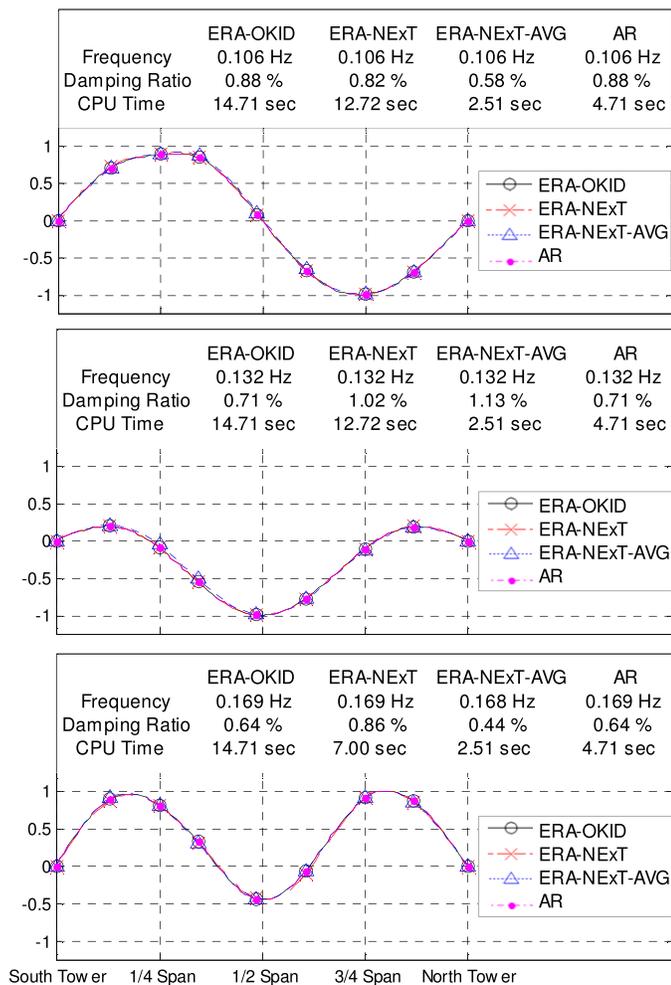


Fig. 10. Comparison of the first three identified vertical modes and the corresponding CPU time for ERA-OKID, ERA-NExT, ERA-NExT-AVG, and AR model

Conclusion

As a global health monitoring measure, a new modal identification method for the output-only systems is proposed, which is based on the ERA-NExT method. The proposed method, called the ERA-NExT-AVG, reduces the size of Markov parameters to formulate a Hankel matrix. The coded average of each row vector of the correlation functions in Hankel matrix tremendously reduces the computational cost of the identification algorithm especially for the SVD and eigenvalue decomposition procedures. Computer simulations are performed for a shear structure subjected to Gaussian white noise. The stabilization diagram and accuracy plots for each mode are used to demonstrate that the proposed method identifies the structural modal parameters accurately. To evaluate the efficiency of the proposed method, its computational cost is compared with the ERA-OKID, ERA-NExT, and AR methods in terms of both the number of operations and CPU time for different model orders. In both comparisons, with a consistently smaller computational cost, reliable modal identification is achieved using the proposed method. Application of the ERA-NExT-AVG to the ambient acceleration data from GGB is used to evaluate its performance for field data by estimating the vertical and torsional modes of the bridge. The identified modes are compared with the results from other stochastic

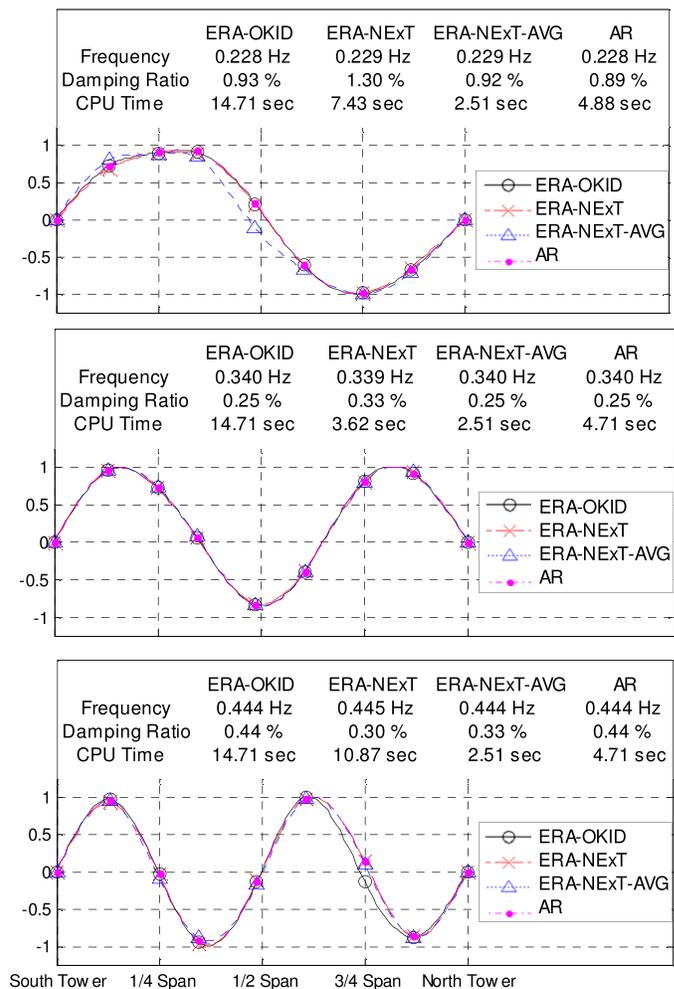


Fig. 11. Comparison of the first three identified torsional modes and the corresponding CPU time for ERA-OKID, ERA-NExT, ERA-NExT-AVG, and AR model

identification methods. The results show that with a smaller CPU time, the ERA-NExT-AVG is able to achieve robust modal identification that is comparable with other methods in accuracy and is consistent with the behavior of the monitored structure.

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