Chapter 18
Modal Parameter Uncertainty Quantification Using PCR Implementation with SMIT

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Abstract The System Identification (SID) techniques for output-only systems, combined with the use of Wireless Sensor Network (WSN), provide many opportunities to monitor large scale civil infrastructure. Recently, research associated with uncertainties in the measurement data has been conducted to quantify the level of noise and to improve the performance of SID methods. This paper presents the effect of measurement noise when the data is used in Eigensystem Realization Algorithm (ERA) based methods including Observer Kalman filter Identification (OKID), Natural Excitation Technique (NExT), and NExT using average scheme (NExT-AVG). Each algorithm estimates impulse response for ERA algorithm differently, which results in different noise level in terms of Physical Contribution Ratio (PCR) and affects the accuracy of identification results. In order to compare the effect of noise from each SID methods, modal parameters are estimated using the numerically simulated response from simply supported beam model and wireless sensor data from Golden Gate Bridge (GGB). All identification procedures are supported by Structural Modal Identification Toolsuite (SMIT) which provides a convenient environment to access various SID methods.

Keywords System identification • Eigensystem realization algorithm • Physical contribution ratio • Structural modal identification toolsuite • Wireless sensor network

18.1 Introduction

System Identification (SID) techniques are widely implemented for global Structural Health Monitoring (SHM) to characterize dynamic modal parameters which can be used for the assessment of structural performance, the quantification of the potential deterioration, and the calibration of finite element models. The development of Wireless Sensor Networks (WSN) increases the demand for efficient and accurate SID for analyzing large scale civil infrastructures. Over the past three decades, efforts to improve the performance of SID have focused on modal parameters accurately, to minimize the costly computation, and to simplify the implementation [1]. In practice, the performance of SID algorithms is affected by the measured/simulated vibration data and the parameters associated with the individual method, which has been studied to construct reliable monitoring systems.

As a pioneering study, Eigensystem Realization Algorithm (ERA) developed and has served for the SID of many engineering systems [2]. ERA uses impulse response and at-rest initial conditions to derive the equivalent state matrix which includes the information of mass, stiffness, and damping coefficient matrices in linear systems. In order to expand the applicability of ERA to the general input/output systems, observer Kalman filter is introduced, which locates the eigenvalues of the observer state matrix on any desired values. The OKID technique is further improved for the output-only systems [3] in which the responses are separated into the deterministic and stochastic sub-systems. Then, the contribution of the deterministic sub-system is assumed to derive the observer state matrix with only output responses. Alternatively, ERA-Natural Excitation Technique (NExT) is also applicable for output-only systems [4]. The numerical expression for NExT shows that the correlation functions for output responses are the linear combination of impulse responses and alternate the Markov parameters in ERA. Auto-Regressive Moving-Average (ARMA) is another time domain method widely used for

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many engineering problems [5, 6]. The canonically arranged input/output responses directly derive transition state matrix. Depending on the type of input, ARMA is further extended to exogenous input/output systems (ARX) [7] and output-only systems (AR) [8]. Stochastic Subspace Identification (SSI) uses the projections of the future responses onto the previous input/output or output responses and shows reliable identification results [9, 10].

Each SID method requires a specific type of data such as impulse response, input/output, or output data. Most methods utilize over-parameterization technique to compensate the rank deficiency and to estimate accurate modal parameters. For these reasons, a comparison study of the performance of these methods is necessary. There is existing literature on the accuracy comparison of SID methods [11, 12]. The computational time and number of operations are estimated as measures of efficiency and conclude that changes in model order, specific modes, and SID method affect the identification results [3, 13].

The presence of uncertainty in vibration response is another issue for modal identification. The measurement noise is considered as an epistemic uncertainty for SID and is possible to increase the identification of computational modes and the distortion of modes [14]. Many studies for sensing devices show that the measurement noise does not have a significant effect on the amplitude of the actual response and can be considered random for modal identification. The effect of the measurement noise depending on the choice of the SID method has been less studied, even though it is critical for accurate estimation of the modal parameters of structural systems.

This paper aims to compare the performance of several SID methods based on ERA for output-only systems (ERA-OKID, ERA-NExT, and ERA-NExT-AVG). In order to investigate the effect of uncertainty in vibration response, Physical Contribution Ratio (PCR) [1] is adopted as a measure of signal power at sensing nodes. Two examples are used to investigate the effect of uncertainty for modal identification: (1) numerically simulated 4-DOF beam model subject to white noise excitation with several levels of measurement noise and (2) ambient vibration, measured from wireless sensors on Golden Gate Bridge.

18.2 Modal Identification Using ERA-Based Methods

Eigensystem Realization Algorithm (ERA) is one of most widely used time-domain system identification methods [2]. ERA uses impulse responses to derive the modal parameters. Consider the state space description of dynamic system in discretized form as:

$$\begin{aligned} x_{i+1} &= Ax_i + Bu_i \\ y_i &= Cx_i + Du_i \end{aligned} \tag{18.1}$$

In Eq. 18.1, $x_i \in \Re^{2N}$ is the state vector, N is the number of DOF, u_i is the input vector, and $y_i \in \Re^m$ is the output vector at m locations on the structure (nodes). The coefficients [A, B, C, D] are called the discretized state, input, output, and feed-through matrices, respectively. The at-rest initial state and unit impulse excitation derive the output response in terms of coefficients matrices as $y_i = CA^{i-1}B$ which is also known as Markov parameters. The consecutive Markov parameters are used to construct Hankel matrix which uses over-parameterizing technique to compensate the rank deficiency for modal parameter estimation as:

$$\begin{aligned} H_{i-1} &= \begin{bmatrix} y_i & y_{i+1} & \cdots & y_{i+q-1} \\ y_{i+1} & y_{i+2} & \cdots & y_{i+q} \\ \vdots & \vdots & \ddots & \vdots \\ y_{i+p-1} & y_{i+p} & \cdots & y_{i+p+q-2} \end{bmatrix} \\ &= \begin{bmatrix} CA^{i-1}B & CA^{i}B & \cdots & CA^{i+q-2}B \\ CA^{i}B & CA^{i+1}B & \cdots & CA^{i+q-1}B \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i+p-2}B & CA^{i+p-1}B & \cdots & CA^{i+p+q-3}B \end{bmatrix} \\ &= P_p A^{i-1} Q_q \ \ (i \geq 1) \end{aligned} \tag{18.2}$$

Singular Value Decomposition (SVD) is applied to H_0 to derive transition state matrix A, which is separated into the block matrices P_p and Q_q as:

$$H_0 = R\Sigma S^{\mathrm{T}} = \begin{bmatrix} R_1 & R_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = P_p Q_q \tag{18.3}$$

In Eq. 18.3, the non-zero singular values are used to construct the possible set of block matrices, $P_p = R_1 \Sigma_1{}^{1/2}$ and $Q_q = \Sigma_1{}^{1/2} S_1{}^T$. Accordingly, the transition state matrix is determined as:

$$A = P_p{}^{-1} H_1 Q_q{}^{-1} \tag{18.4}$$

The eigenvalue decomposition of A is used to obtain the modal parameters of the system assuming that the physical system is equivalent to the numerical model. The essence of ERA based SID method is to create the impulse response from the measured vibrations, to assemble unique Hankel matrices depending on the model order specification.

18.2.1 ERA-NExT

Natural Excitation Technique (NExT) algorithm uses correlation functions to identify modal parameter for output-only systems [4]. The numerical expression for the auto and cross correlation function infers the combination of impulse responses. The correlation function $R_{ij,k}$ between response at nodes i and j in terms of time lag k is expressed as:

$$R_{ij,k} = \sum_{r=1}^{N} \left[\frac{\phi_i^r Q_j^r}{m^r \omega_d{}^r} \exp\left(-\zeta^r \omega_n{}^r k\right) \sin\left(\omega_d{}^r k + \theta_r\right) \right] \tag{18.5}$$

In Eq. 18.5, the superscript r denotes a particular mode from a total of N modes; ϕ_i^r is the ith ordinate of the rth mode shape; m^r is the rth modal mass, Q_j^r is a constant associated with response at node j; ζ^r and $\omega_n{}^r$ are the rth mode damping ratio and natural frequency, respectively; $\omega_d{}^r = \omega_n{}^r \sqrt{1 - (\zeta^r)^2}$ is the rth mode damped natural frequency; and θ_r is the phase angle associated with the rth modal response. The sequentially arranged Markov parameters $R_k \in \Re^{m \times m}$ substitute the impulse responses in Eq. 18.2.

18.2.2 ERA-NExT-AVG

A modified NExT algorithm, known as ERA-NExT-AVG, is introduced in [13], which uses a coded average of row vectors in each Markov parameter for original ERA-NExT. The coding coefficient vector is designed to avoid eliminating the special cases for which the simple average results in zero. The element of coded correlation functions $\widehat{R}_k \in \Re^{m \times 1}$ is determined as:

$$\widehat{R}_{i,k} = \frac{1}{m} \sum_{j=1}^{m} \alpha_j R_{ij,k} = \sum_{r=1}^{N} \left[\left(\frac{1}{m} \sum_{j=1}^{m} \alpha_j Q_j^r \right) \frac{\phi_i^r}{m^r \omega_d{}^r} \exp\left(-\zeta^r \omega_n{}^r k\right) \sin\left(\omega_d{}^r k + \theta_r\right) \right] \tag{18.6}$$

In Eq. 18.6, the subscript i is varied from 1 to m, and α_j is a coding coefficient associated with the jth column of the correlation function R_k. The coded correlation functions $\widehat{R}_k$ substitute the impulse response in ERA and the rest procedures follow ERA method. The main advantage of ERA-NExT-AVG is to reduce the computational cost by reducing the size of Markov parameter while maintaining an accuracy equivalent with ERA-NExT.

18.2.3 ERA-OKID-OO

The observer Kalman filter is used to remedy the lack of initial conditions in ERA method and extends its applicability to the general input/output systems [3]. The method is further extended to the identification of output only systems by separating the response to the sum of deterministic and stochastic contributions. For the structural systems experiencing non-measurable noise input, it can be assumed that the stochastic response governs the entire system and the output responses are represented as a function of $[A,C]$. The observer gain My_i is introduced to Eq. 18.1 and the output response is observed as:

$$y_i = C\overline{A}^i x_0 + \sum_{\tau=0}^{i-1} \widehat{Y}_\tau y_{i-\tau-1} \tag{18.7}$$

In Eq. 18.7, $\widehat{Y}_\tau = -C\overline{A}^\tau M = -C(A+MC)^\tau M$, and $\overline{A}^i$ approaches zero for $i \geq p$ where p is the number of independent Markov parameters. The Markov parameters for ERA-OKID-OO are determined as $Y_\tau = CA^\tau M$, which are calculated using the recursive relationship of auto-regressive coefficients $\widehat{Y}_\tau$.

18.3 Physical Contribution Ratio

The Physical Contribution Ratio (PCR) is introduced to quantify the participation of modal vibration when modal parameters are estimated from the impulse response [1]. ERA based SID methods commonly formulate the Hankel matrix composed of sequentially arranged Markov parameters (impulse responses) and estimate the equivalent numerical model using Singular Value Decomposition (SVD). Assuming that the Markov parameters include noise contribution, the effect is directly shown in discretized state-space matrices.

Markov parameters ($y_i = CA^{i-1}B$) in Eq. 18.2 are transformed into the modal coordinates from physical coordinates using the eigenvalue decomposition of state matrix as:

$$y_i = C\psi\Lambda^{i-1}\psi^{-1}B \tag{18.8}$$

In Eq. 18.8, ψ and Λ are the matrices of eigenvectors and eigenvalues of A. Accordingly, the mode shape and modal amplitude matrixes are determined as:

$$C\psi = \begin{bmatrix}\phi_1 \ \phi_2 \cdots \phi_m\end{bmatrix} \quad \text{and} \quad \psi^{-1}B = \begin{bmatrix}{b_1}^T \ {b_2}^T \cdots {b_m}^T\end{bmatrix}^T \tag{18.9}$$

In Eq. 18.9, ϕ_1 to ϕ_m are the column vectors of identified mode shapes and b_1 to b_m are the row vectors of transformed impulse at each node; m denotes the number of identified modes including noise parameters. Due to the over-parameterization, the results of SID normally involve large amount of spurious modes that can be removed using stabilization diagram and expert judgments [15]. In this study, the number of identified modes m is conservatively determined as ten times larger than true structural modes in order not to lose true modes with small PCR values.

Expanding the estimated impulse response, the PCR of the jth mode in the kth diagonal component is defined as [1].

$$\text{PCR}_{kj} = \frac{\sum_n \phi_{kj} b_{jk} {\lambda_j}^{n-1}}{\sum_m \sum_{i=1}^{m} \phi_{ki} b_{ik} {\lambda_i}^{n-1}} \tag{18.10}$$

In Eq. 18.10, n is the time index and λ_j is the jth eigenvalue of A. The PCR quantifies the contribution of true modes over time, indicating a measure of the signal power in true structural modes amongst estimated impulse responses. Since the measurement noise is decentralized with reference to all the modes, the increase of noise contamination result in the decrease in PCR values.

18.4 Validation Using Measured Data

Two examples are used to validate the performance of PCR depending on the identification methods: (1) numerically simulated output data from a 4-DOF simply supported beam model and (2) ambient vibration data from Golden Gate Bridge (GGB). Based on the exact modal information in numerically simulated model, the relationship between estimated PCR values and errors in identified modal parameters is investigated. The GGB example validates the use of PCR for practical implementation to achieve accurate modal identification.

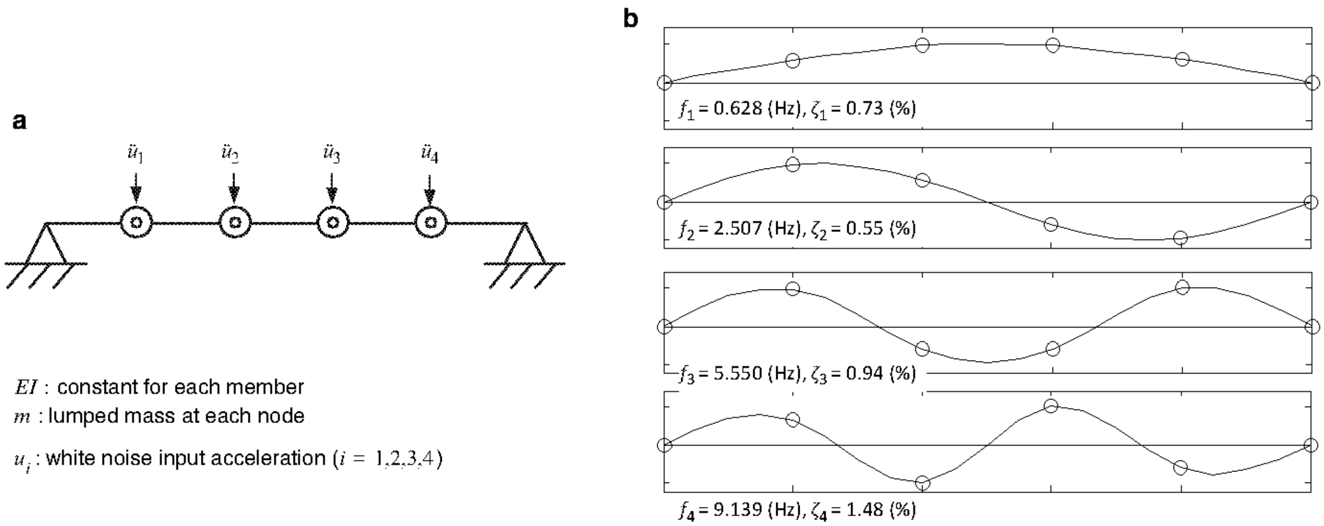

Fig. 18.1 (**a**) 4-DOF simply supported beam model, (**b**) identified mode shapes with exact natural frequencies and damping ratios

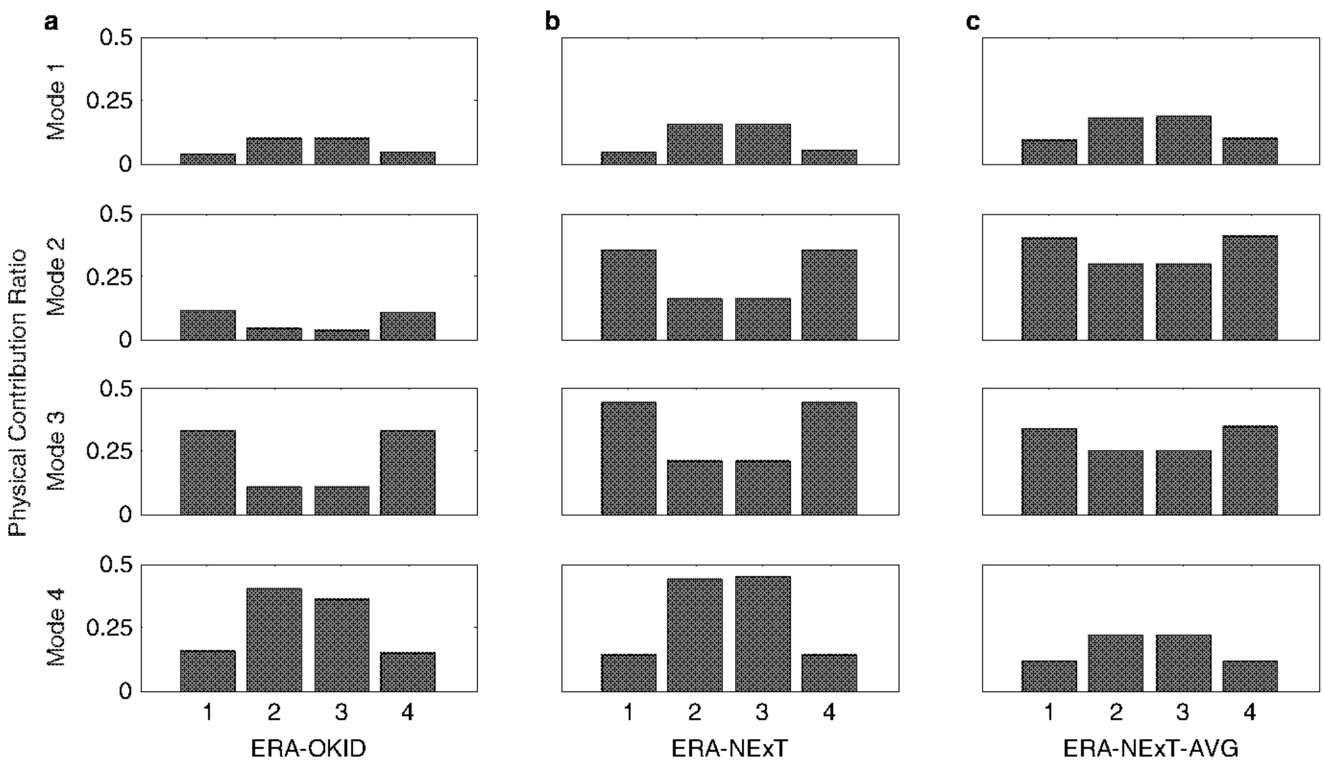

Fig. 18.2 PCR values at node locations for each mode

18.4.1 Simulated Simply Supported Beam

A 4-DOF simply supported beam is designed to simulate the input/output data as shown in Fig. 18.1. The mass is lumped on each DOF and the damping coefficient matrix is formulated as the linear combination of mass and stiffness (classical damping). A set of output response with frequency of 200 Hz is simulated, subjected to white noise excitation at each node. The model order of 100, which is sufficient to estimate all structural modes, is chosen and 40 modes based on the contribution to Hankel matrix are used to calculate the PCR values.

Figure 18.2 shows the PCR values at sensing nodes depending on modes and SID methods. In general, the shape of PCR is similar to the absolute values of mode shape vectors and the smaller PCR values are observed from the lower modes for

Table 18.1 Normalized error in identified modal parameters

Error comparison (%)	ERA-OKID				ERA-NExT				ERA-NExT-AVG			
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 1	Mode 2	Mode 3	Mode 4	Mode 1	Mode 2	Mode 3	Mode 4
Frequency	2.41	0.13	0.12	0.67	0.81	0.14	0.06	0.11	0.68	0.05	0.06	0.36
Damping ratio	493	63.2	20.6	5.11	394	581	12.3	227	17.8	15.3	94.0	24.2
MAC w/exact mode	2.14E−5	2.19E−4	3.53E−3	4.25E−3	2.41E−4	4.75E−5	4.19E−3	0.10	2.82E−6	3.18E−4	2.65E−2	1.51E−2
MAC w/MPC	1.59	2.12	2.76	1.49	3.09	0.70	0.58	3.53	0.27	0.31	0.39	0.2

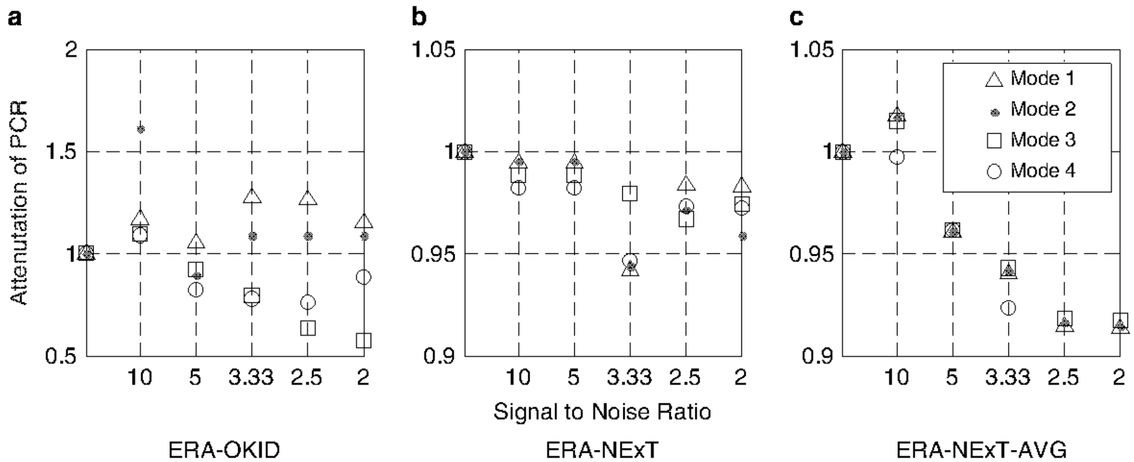

Fig. 18.3 Variation of the PCR values due to increase of simulated noise

all methods. Since the ambient excitation is applied on each node independently, a small contribution is observed for the first mode. Especially for ERA-OKID records, relatively smaller contributions for the first and second modes are observed, which indicate that the estimated impulse response does not include much contribution of these modes compared to other methods.

By comparison of the accuracy in identified modal parameters, further results are available. Table 18.1 shows the percentage changes of accuracy levels, which are estimated by taking normalized difference ratio for modal frequency and damping ratio and calculating the error in MAC between identified modal parameters and exact values. The detail of error evaluation is explained in [3]. Additionally, the MAC between identified mode and PCR is included in this table. The first two modes for ERA-OKID show 2.41 and 0.13 % for modal frequencies and more than 200 and 63.19 % for damping ratios when model order of 100 is used. These errors are significantly larger than ERA-NExT and ERA-NExT-AVG while the errors in third and fourth modes are similar in all methods. The shapes of PCR values can be used as a measure of accuracy in mode shape estimation. For example, the fourth mode where ERA-NExT overestimates the contribution for node 2 and 3 compared to other methods and exact mode shape. The accuracy comparison also shows that the relative error (%) in damping ratios are [5.11 227 24.2] for [ERA-OKID, ERA-NExT, ERA-NExT-AVG] whereas the other errors are much smaller. This comparison study verifies that the information in the estimated impulse response is highly correlated with the accuracy of SID results, and a high level of PCR produces the accurate modal parameter estimates.

In order to investigate the effect of uncertainty more specifically, an artificial noise is simulated and added to the original response and the percentage changes in PCR is observed (Fig. 18.3). In general, the attenuation of PCR is more pronounced for ERA-NExT and ERA-NExT-AVG while the PCR values for first and second modes fluctuate with the increase of noise contamination. Similar to the previous results, this result also supports that the impulse response estimated by ERA-OKID includes small contribution for first and second modes, which results in inaccurate modal parameters estimation, especially for damping ratios.

18.4.2 Ambient Vibration Data from Golden Gate Bridge (GGB)

In this section, a set of ambient vibration response from Golden Gate Bridge (GGB) is used to quantify PCR and accurate modal parameter estimation. The acceleration data were measured using 65 wireless sensing nodes and used to identify 25

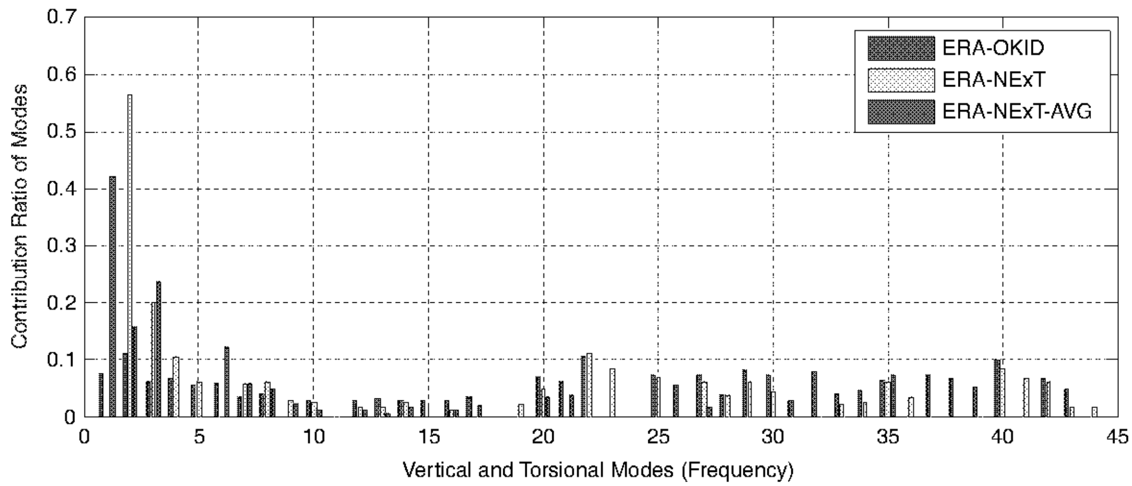

Fig. 18.4 Overall PCR of 25 vertical and 19 torsional modes of GGB

vertical, 19 torsional, and 23 transverse modes from the main span of the bridge [16–18]. For this example, the vertical vibration responses from seven sensors, almost uniformly located on the west side and three sensors on the east side of the bridge are used to quantify PCR values for the total of 44 modes (vertical and torsional). The vibration responses are passed through a low pass filter with a cutoff frequency of 5 Hz and downsampled accordingly and the model order of 100 is used for investigated SID methods.

The square sum of PCR values at sensor nodes for each mode is obtained to quantify the modal contribution precisely (Fig. 18.4). Similar to the simply supported beam example, a relatively small contribution is observed for low modes when ERA-OKID is implemented. However, ERA-OKID shows reliable PCR estimates for higher modes. Overall, the estimated impulse response for ERA-OKID distributes the modal contribution evenly in all modes. Therefore, the number of identified modes is larger than ERA-NExT and ERA-NExT-AVG when the same model order is applied, which is also noted in the literatures [3, 13]. The coding coefficient for ERA-NExT-AVG avoids eliminating true structural modes and enhances the true mode contributions, so that first mode is successfully identified even though ERA-NExT fails to identify this mode. However, due to the lack of information for higher modes, the coding coefficients cause negative effect for PCR values and are unable to identify true structural modes in some cases.

It is observed that for the 2nd, 3rd, and 12th modes, the highest peaks of PCR are reported in ERA-NExT, ERA-NExT-AVG, and ERA-OKID, respectively. Figure 18.5 illustrates the identified mode shapes for these modes with identified modal frequency and damping ratio recorded for each mode shape. The MAC values between identified mode shape and PCR distribution also highly correlated and can be used to eliminate inaccurate estimation.

18.5 Conclusion

This study aims to evaluate the effect of measurement noise for modal parameter estimation. For ERA-based SID methods, the estimation of impulse response (Markov parameters) is a common procedure. The identification results vary depending on the level of noise in the estimated impulse response. The numerically simulated 4-DOF beam model is used to quantify the effect of measurement noise in terms of PCR. The simulated noise, added to the original response, shows the attenuation of PCR in the estimated impulse response. The decreasing trend is monotonic for ERA-NExT and ERA-NExT-AVG. The PCR for first two modes are random for ERA-OKID, which directly contributes to significant errors in estimating damping ratios. The accuracy of identified modal parameters is highly correlated to the level of PCR.

The PCR comparison is conducted on the identified modal parameters from ambient vibrations of Golden Gate Bridge. The estimates of PCR for each mode indicate that ERA-OKID evenly distributes the modal contributions whereas ERA-NExT and ERA-NExT-AVG show high levels of PCR for low modes. Although ERA-NExT-AVG is computationally efficient and accurate, it is less effective in identifying high modes. The MAC comparison between an identified mode shape and PCR show that it can be used as a threshold to determine true modes from the noise parameters.

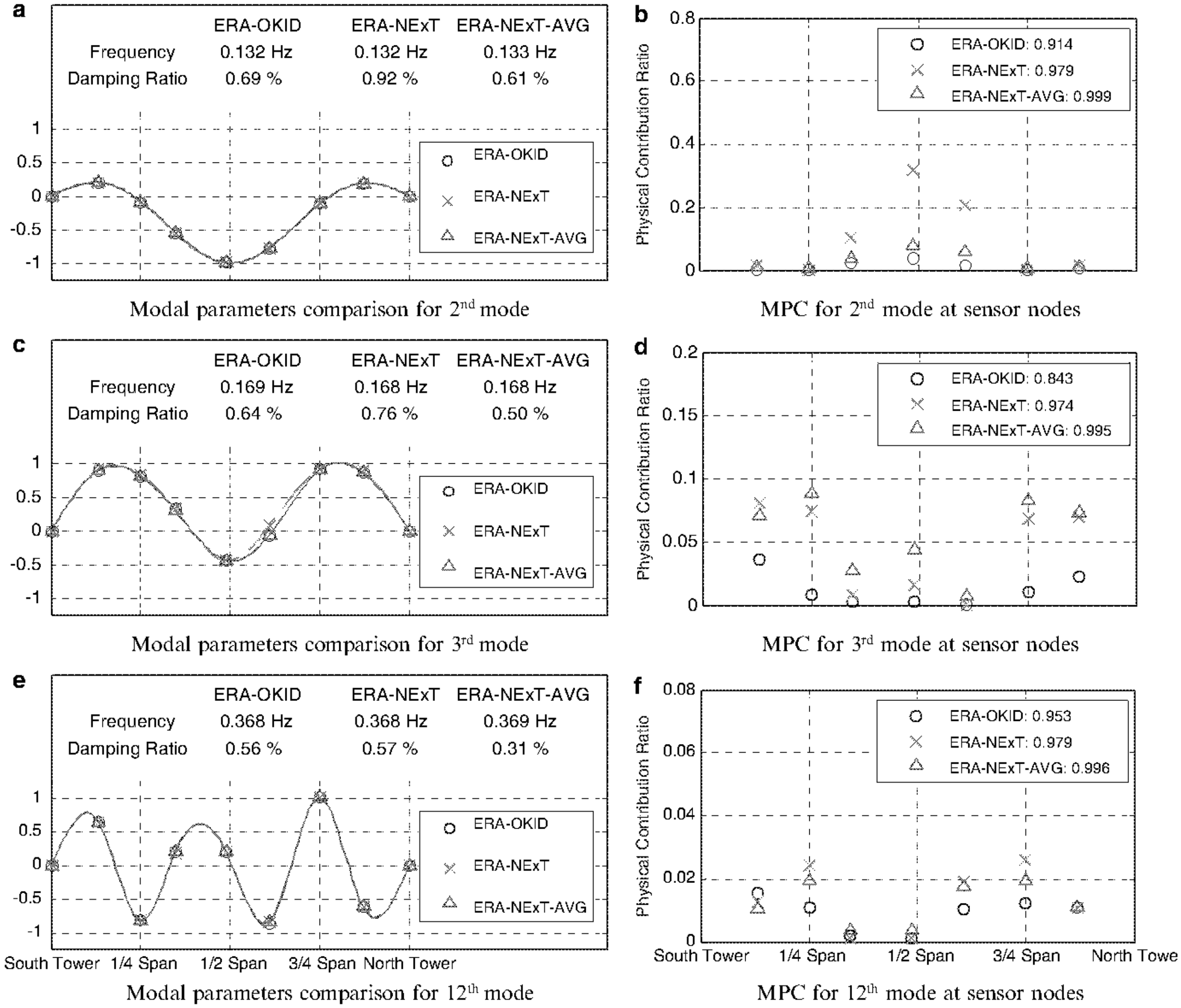

Fig. 18.5 Identified modal parameter comparison and PCR values

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