# Chapter 23
# Iterative Modal Identification Algorithm; Implementation and Evaluation

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**Abstract** Applying Wireless sensor networks (WSNs) in structural health monitoring (SHM) systems has received significant interest from research communities in recent years. While incorporating wireless technology in monitoring systems has provided considerable improvements, new approaches are still needed to address existing challenges in their application. A major challenge in application of WSNs in SHM is the limited power resources of wireless sensors and the latency in the data processing due to the low communication bandwidth. The performance of the networks on both of these factors can be improved through the use of an iterative modal identification algorithm, called IMID. This algorithm uses the local processing capability of wireless sensors and provides a substantial reduction in required communication for identification of system's modal properties. The iterative approach is such that, starting from an initial estimate of the system's parameter, all of the nodes of the network use their local measurement and update the estimated modal parameters one-by-one until the convergence of results happens. In this manner, the system's parameters are the only data that need to be transmitted through the network for updating. This approach results in the significant reduction in the total volume of communication. This paper presents the application of the IMID in modal identification of a 3-D steel truss structure. The results are discussed and the performance of the algorithm is evaluated.

**Keywords** Wireless sensor networks • Structural health monitoring • Iterative modal identification

## 23.1 Introduction

Identification of dynamic parameters of structural systems through vibration monitoring is important for the assessment of structural performance and calibration of finite element models. Over the past few years, vibration monitoring systems have significantly improved by incorporating the advancements in sensing and communication technology. One of the distinct improvements in the vibration monitoring is application of wireless technology for communication in the sensing networks. Using Wireless Sensor Networks (WSNs), the prohibitive costs associated with wiring is eliminated and the instrumentation of the network becomes considerably more affordable. At the same time, each sensor in a WSN is integrated with a low-power microcontroller which enables local data processing task in the sensing network. As the advantages of WSN became evident, researchers developed different WSN platforms and deployed in Structural Health Monitoring (SHM) systems [1–3].

Utilizing WSNs in health monitoring of infrastructure raised the demand of having scalable sensing networks, functional in long term scenarios. The major challenges in scalability of WSNs are limited communication bandwidth and the finite power resource at remote nodes. One of the most power consuming tasks among all the tasks, performed at the sensor nodes, is data communication. Realizing the fact that the on-board computation consumes significantly less energy compared to communication, researchers focused on strategies in which the computational core of the sensors contributes in data processing tasks, aiming to minimize the communication. These approaches assist with both preserving the limited available power at remote nodes and the communication bandwidth. The basic idea in distributing computational task within the sensing network is to perform a portion of data interrogation locally and communicating the more informative data to the base station.

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Use of embedded algorithms on remote sensors started with basic approaches such as filtering, data compression [4] and conversion of the data from time to frequency domain [5]. Numerous researchers also developed distributed algorithms for detecting and quantifying damage in the instrumented structure [6–8]. While distributed algorithms have been successful in data interrogation and, to some extent, in damage detection, they are still lacking in the implementation of modal identification techniques. The challenge in distributing modal identification process among remote sensors is the need for having spatial information at central computational core. A number of researchers studied this problem and proposed methodologies to address this challenge. One of the early proposed approaches was implementation of peak picking algorithm and transmission of imaginary components of Fourier spectra at modal peaks, thus capturing system's poles with their phase information [9]. A family of techniques involving hierarchical network topology has been also presented in the literature [10–12] in which the network is divided into a number of overlapped sub-networks with cluster heads. The basic idea in these approaches is to transmit data from cluster heads to the rest of the cluster nodes (called leaf nodes), local calculation of correlation functions, and at the end, transmission of the correlation functions from sub-networks to the base station for estimation of global modal properties. Another proposed approach for distribution of modal identification algorithms is the use of Regularized Auto Regressive (AR) Models [13]. In this method a restriction is applied on AR parameters of the Multivariate AR models which eliminates the computation of correlation between signals measured at nodes that are far apart, thus reducing the volume of data passed through the network, particularly when multi-hop data transmission is applied.

While proposed approaches reduce the amount of communicated data in specified architectures, they are restricted by the underlying identification algorithms. Recently, a new approach proposed on this topic in which the modal properties of a system are identified in an iterative method [14]. This method, called Iterative modal Identification (IMID) relies on an iterative estimation method which estimates unknown parameters in the absence of complete information about the system. IMID assigns the computational task of modal identification to each remote node and limits the data communication to transmission of only modal analysis results. A basic assumption in implementation of the algorithm was availability of system's input function at remote nodes of the network. Evidently, such an assumption would be challenging in deployment of the algorithm on large scale, real structures. In this paper, IMID algorithm is adapted to work with the output data while the input is impulse. The algorithm is deployed on a set of data collected from a truss structure to identify the modal properties iteratively. The obtained results validate the performance of the algorithm and the convergence of the iteration. The modal properties of the truss are also estimated by traditional centralized modal identification algorithm and are used as a reference for validation of results from IMID. The obtained results are discussed and the significance of the approach is evaluated in the paper.

## 23.2 Iterative Modal Identification, Methodology

Implementation of most of the existing modal identification algorithms requires the access to the entire measured data. This requirement is the main restriction in developing distributed algorithm for WSNs. The advantage of IMID is that it estimates the modal parameters of the system without requiring simultaneous access to the entire data. The technique which is used in this algorithm relies on a class of estimation algorithm, called Expectation-Maximization (EM). EM estimates unknown parameter ($\theta$), given the measurement data (Y) in the presence of some hidden data ($\hat{Y}$), or in other words, in the presence of incomplete data [15].

Considering the log-likelihood function of unknown parameters $\theta$ as:

$$L(\theta) = \log(p(Y/\theta)) \tag{23.1}$$

the estimation of unknowns ($\theta$) is given by maximizing the function, L($\theta$), over $\theta$:

$$\theta = Arg.\max(L(\theta)) \tag{23.2}$$

where Y is the available data (complete measured data).

Now considering the case that the entire data for estimation is not available, the EM becomes applicable. EM first estimates the complete data using assumed parameters, $\theta_p$, (expectation phase), then maximizes the likelihood function over system's parameter to find $\theta_{p+1}$ (maximization step) and continues until the convergence for the parameters is achieved. The concept of EM is also illustrated in Fig. 23.1.

The presence of incomplete data in EM is similar to situation when each remote node has access to its own collected data, and the data from the rest of the sensors are not available. IMID offers that each node separately estimates the system's

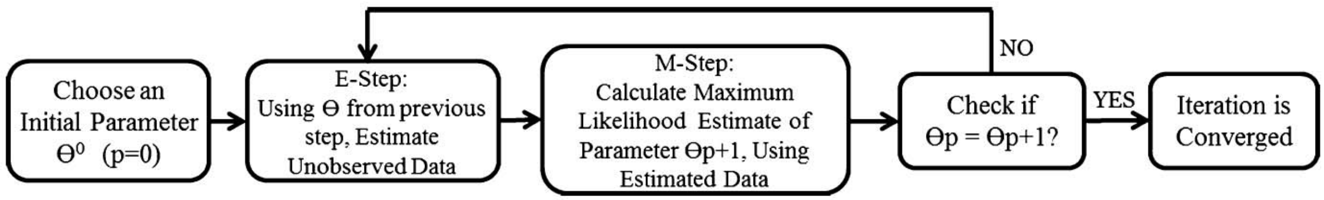


**Fig. 23.1** The block chart presenting the Expectation-Maximization algorithm

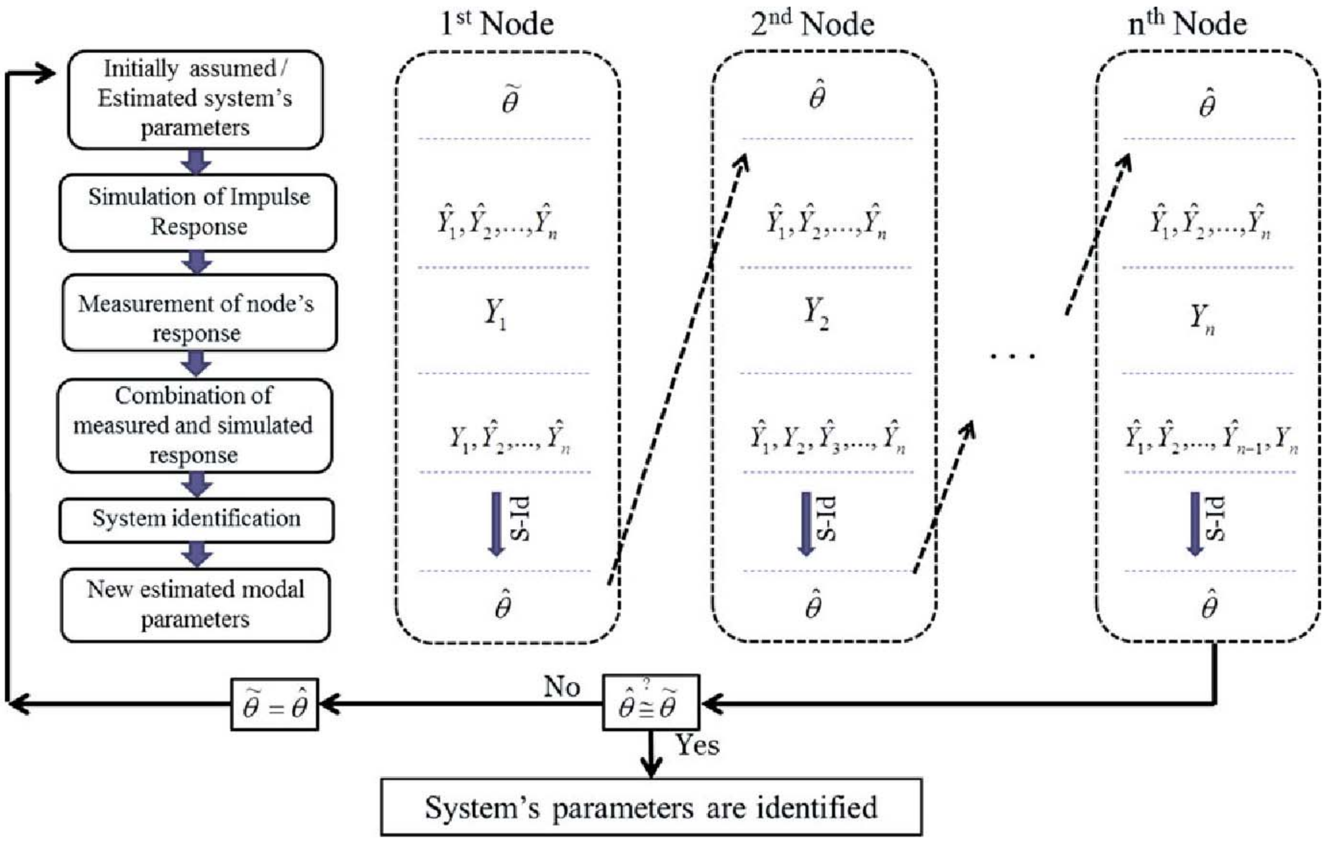


**Fig. 23.2** The block chart presenting the iterative modal identification algorithm

parameter ($\theta$) based on its observation (measured data), $Y$, and the assumed responses in the other nodes, $\hat{Y}$. An initial estimate of system's parameters is used for making assumption on the response at other nodes.

Considering the implementation of IMID on a structure instrumented by $N$ sensor nodes, the first node uses the initial value of the parameters and makes a numerical simulation to estimate the response on other nodes (expectation step in EM algorithm). The simulated response and the measured data at this node are used to make an estimate on system's modal parameters (maximization step in EM algorithm). Then this node sends the system's parameters to the next node in the network for a similar local processing. The same steps are taken in all of the nodes of the network one-by-one. However, for simulation of the response, each node uses the system's parameter estimated at its previous node. Therefore, each node influences on the estimation by its measured data. This procedure is continued inside the network until the estimated parameters are converged. Note that as a requirement for implementation of the algorithm, an initial estimation of system's parameters should be available.

Considering the fact that the size of estimated parameters is very small, compared to that of the time history data, depending on the number of iteration cycles, IMID can significantly reduce the communication burden of the network. Figure 23.2, shows a step-by-step block chart, illustrating IMID.

The two steps in IMID are (1): simulation of the response using the estimated parameters, and (2): modal identification using the collected data and simulated response. A variety of algorithms is available for both simulation and modal identification (e.g. Newmark and Central Difference numerical methods for simulation and Eigensystem Realization Algorithm (ERA), Stochastic Subspace Identification (SSI), and ARX algorithm for modal identification).

One of the valid models for characterizing linear systems is Auto Regressive with Exogenous (ARX) model. ARX can be used for both simulation and identification steps. The ARX model can be written as:

$$\sum_{i=0}^{p} \alpha_i y(n-i) = \sum_{i=1}^{q} \beta_i u(n-i) + e(n) \tag{23.3}$$

where $y = [y_1(n)\ y_2(n)\ \ldots\ y_m(n)]$ and $u(n) = [u_1(n)\ u_2(n)\ \ldots\ u_r(n)]$ are matrices including output and input vectors respectively. $\alpha_i$'s and $\beta_i$'s are ARX coefficients, $e(n)$ represents the noise and measurement error, and $p$ and $q$ are orders of the autoregressive and exogenous parts of the ARX model. For IMID based on ARX model, ARX parameters are considered to represent the structural system and therefore, these parameters are communicated throughout the network.

Having the ARX parameters of the system, response at any time step can be estimated based on the past inputs and outputs and the current input. Considering the noise has zero mean, (23.3) can be rewritten to estimate for the response at time step $n$:

$$\bar{y}(n) = -\sum_{i=1}^{p} \alpha_i y(n-i) + \sum_{i=0}^{q} \beta_i x(n-i) \tag{23.4}$$

Now considering the input is impulse function, for $n > p + 1$, (23.4) will be simplified to:

$$\bar{y}(n) = -\sum_{i=1}^{p} \alpha_i y(n-i) \tag{23.5}$$

This equation can be used for one-step output prediction. For multi-step output prediction, the vector of $s$ steps responses can be obtained directly from following relationship [16]:

$$\begin{bmatrix} y(n) \\ y(n+1) \\ \vdots \\ y(n+s-1) \end{bmatrix} = -\begin{bmatrix} \alpha_p & \alpha_{p-1} & \cdots & \alpha_1 \\ \alpha_p^{(1)} & \alpha_{p-1}^{(1)} & \cdots & \alpha_1^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_p^{(s-1)} & \alpha_{p-1}^{(s-1)} & \cdots & \alpha_1^{(s-1)} \end{bmatrix} \cdot \begin{bmatrix} y(n-p) \\ y(n-p+1) \\ \vdots \\ y(n-1) \end{bmatrix} \tag{23.6}$$

where:

$$\alpha_1^{(0)} = \alpha_1$$

$$\alpha_1^{(k)} = \alpha_{k+1} - \sum_{i=1}^{k} \alpha_i \alpha_1^{(k-i)} \quad \textit{for} \quad k < p$$

$$\alpha_1^{(k)} = -\sum_{i=1}^{p} \alpha_i \alpha_1^{(k-i)} \qquad \textit{for} \quad k \geq p$$

Therefore, having the AR parameters of the system, and knowing that the system is excited by impulse, the response can be easily simulated.

The second major step in IMID is identification step. The main goal in the identification step is to estimate the system's AR parameters. However, to check the convergence, it is required to also extract the modal properties of the system. To this end, the AR model can be rearranged to the state space representation for modal identification purpose. Consider the state space model as:

$$x(n+1) = A_d x(n) + B_d u(n) \tag{23.7}$$

$$y(n) = Cx(n) + Du(n) \tag{23.8}$$

where $x(n)$ is the state vector, $y(n)$ is the observation vector, and $u(n)$ is the input vector at time step $n$; $A_d$ is the state and $B_d$ is input matrices in discrete format, $C$ is the observation matrix and $D$ is transmission matrix. Choosing the state vector as:

$$x(n) = [\,y(n) \quad y(n+1) \quad \cdots \quad y(n+k-1)]'$$

the state matrix can be expressed in the controller form as:

$$A_d = \begin{bmatrix} 0 & I & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I \\ -\alpha_p & -\alpha_{p-1} & \cdots & -\alpha_1 \end{bmatrix} \tag{23.9}$$

and the observation matrix as:

$$C = [\,I \quad 0 \quad . \quad . \quad 0 \quad 0\,] \tag{23.10}$$

where [$I$] and [$0$] are identity and zero matrices with appropriate dimensions, respectively. Eigenvalue decomposition of the state matrix ($A_d$) results in the matrices of eigenvalues ($\lambda_i$'s) and eigenvectors ($\psi_i$'s) from which the natural frequencies, damping ratios and mode shapes of the system can be obtained using following relationships:

$$\lambda_{c_i}, \lambda_{c_i}^* = -\zeta_i\omega_i \pm j\omega_i\sqrt{1-\zeta_i^2} \tag{23.11}$$

$$\phi_i = C\Psi_i \tag{23.12}$$

where $\omega_i$ and $\zeta_i$ are natural frequencies and damping ratios and $\lambda_c$ is the eigenvalue of the continuous state matrix. $\lambda_c$ can be obtained from this equation:

$$\lambda_{c_i} = \frac{\ln(\lambda_i)}{\Delta t} \tag{23.13}$$

where $\Delta t$ is the sampling time.

Estimating these modal properties at each node, the convergence of the IMID is assessed and when these parameters are stabilized, the iteration can be stopped and the final results will be sent to the base station. This way, no further process is required at the base station since the modal properties are already estimated.

## 23.3 Implementation on a Truss Structure

To validate the IMID when the input is impulse, the algorithm is applied in modal identification of a three dimensional steel truss structure. The truss has 27.5 ft length and six panels as shown in Fig. 23.3. A network of 10 wireless sensors is installed on lower chord of the truss to collect the acceleration data in lateral and vertical directions. Figure 23.3b shows the configuration of sensor network on the truss. The accelerometers are LIS3L02AS4 [17] with 50 μg/√Hz noise density in X and Y direction and resolution of 0.66 V/g, capturing acceleration in ±2 g range.

Having a relatively stiff structure with natural frequencies all above 10 Hz, a relatively long response measurement with a high sampling rate is required for obtaining fundamental natural modes. For the experiments, the impacts were applied on location 2 (Fig. 23.3b) in both vertical and lateral directions. It takes about 30–40 s to have the impulse response with a peak of 0.6 g fully attenuated. The sampling rate was 280 Hz and the length of the data used for the algorithm was 10,000 points. Figure 23.4 shows the collected response from sensor number 3 at the mid-span and its power spectrum. Using AR algorithm, the modal frequencies, damping ratios and mode shapes of the truss are identified to be used as a reference for validation of the results, obtained from IMID implementation. Figure 23.5 shows the identified modal properties.

To begin the iteration process, an initial estimation of the structural model is required as the starting point of the iterative process. For this purpose, random masses are added to different nodes such that the additional masses make about 10% changes into the natural frequencies of the system. An AR model is fitted to the response of the altered truss to represent the initial estimate of the structure.

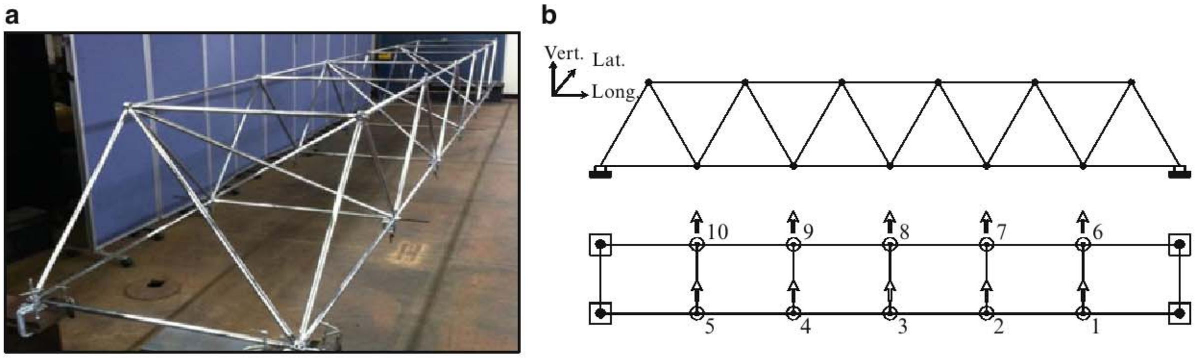


**Fig. 23.3** (**a**) The truss structure, (**b**) the location of sensors on the truss

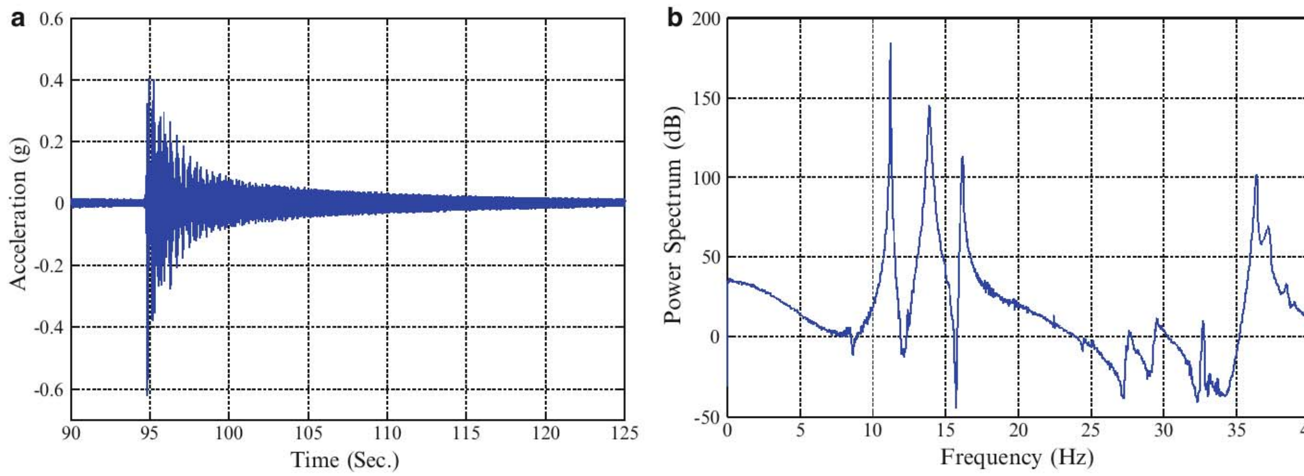


**Fig. 23.4** (**a**) The impulse response at mid-span, (**b**) the power spectrum of the response

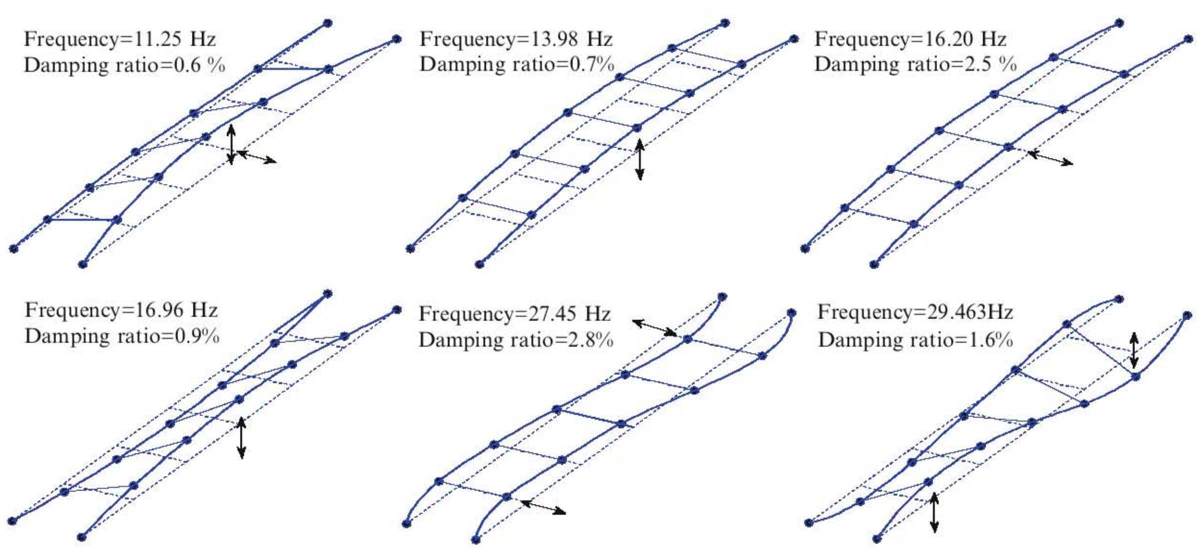


**Fig. 23.5** Fundamental identified modes of the truss structure

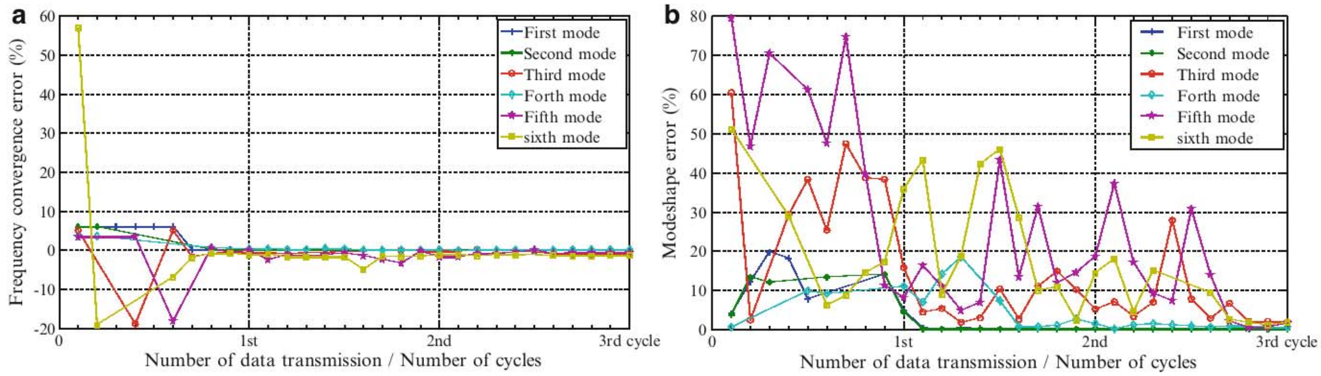


**Fig. 23.6** (**a**) Frequency convergence error, (**b**) the mode shapes convergence error

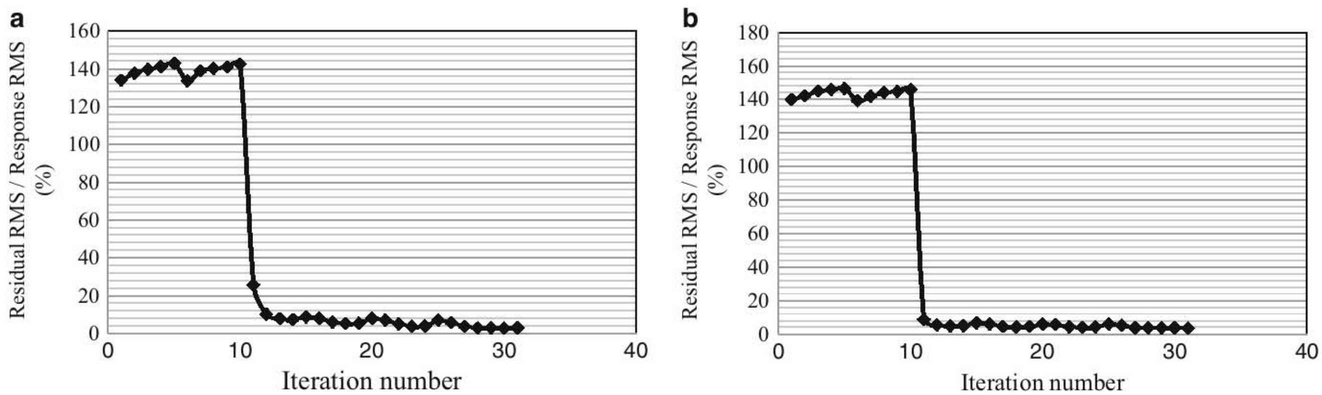


**Fig. 23.7** Residual to response ratio during iteration at different nodes, (**a**) lateral direction (**b**) vertical direction

An important parameter in IMID is the order of the AR model. Usually, higher model order result in better representation of the fitted model which particularly enhance the accuracy in simulation step. However since the AR parameters should be transmitted through the network, increasing the model order increases the size of communication burden in the network. The selected model order for this implementation is 10 which results in 10 matrices of AR parameters, communicated among the nodes of the network.

To check the convergence of the results, identified frequencies and mode shapes are estimated at each node, having the estimated AR parameters received from previous node. Then these modal properties are compared to those estimated from the parameters updated at the node. The criterion for convergence of modal properties can be defined by a predetermined threshold for the changes in the parameters. Modal Accuracy Criterion (MAC) value is used for comparison of mode shapes in consecutive cycles. This criterion is defined as:

$$MAC = \frac{\left(\phi_p^T \times \phi_{p+1}\right)^2}{\left(\phi_p^T \times \phi_p\right) \times \left(\phi_{p+1}^T \times \phi_{p+1}\right)} \tag{23.14}$$

where $\phi_p$ and $\phi_{p+1}$ are the estimated mode shapes at iteration cycles $p$ and $p + 1$.

Figure 23.6a, b shows the error percentage in estimated natural frequencies and mode shapes, versus the iteration cycles. Another parameter which shows the convergence of the procedure is the residual to response ratio. Available at each node are the measured response and the simulated response at the node location. The difference of these two signals defines the residual signal. The ratio of the residual-to-response RMS can be also considered as a parameter for assessing the convergence status. Figure 23.7a, b present this ratio for lateral and vertical directions over the iteration cycles. It can be seen that this ratio drops after one full cycle (passed through all 10 nodes) and becomes relatively small before the third full cycle.

From both Figs. 23.6 and 23.7, it is realized that before three full cycles the natural frequencies and mode shapes variation is less than 2% and the iteration is almost converged. Considering the three cycles of iteration, the total data points need to be communicated through the network are:

$$N_{\text{total}} = 3 \times 10(10 \times 100) = 30000$$

which is significantly less than the required communication in centralized network (less than 30%, considering collection of 10,000 samples from each sensor). Comparing to the previous implementation of the IMID algorithm, this experiment resulted in higher communication burden. However, this implementation does not need availability of input function at remote nodes, thus more applicable in structural monitoring scenarios.

## 23.4 Conclusion

This paper presents deployment of a distributed modal identification algorithm, called IMID, for estimation of modal properties of a 3-D truss structure. In this algorithm the modal properties are expected to be estimated iteratively and locally in the sensing nodes of a network. However in this work, the algorithm is applied on a set of collected data, from a laboratory test on the truss structure. A network of 10 wireless sensors is deployed on the lower chord of the truss to capture the acceleration in vertical and lateral directions.

One of the requirements of the IMID is to have access to the input function for simulating the response of the estimated model. However in this implementation the input is impulse, thus no information about the input is required for simulation of the response. The obtained results showed the convergence of estimated fundamental modes of the structure before three full cycles of iteration. AR algorithm is used for simulation and identification steps and AR parameters are passed through the iteration cycles to represent the updated structural model. However, to check the convergence, modal properties are extracted at each cycle. As an additional parameter for checking the convergence, residual-to-response RMS ratio is defined and plotted over the iterations. A relatively sharp drop after one cycle of iteration and attenuation of the ratio is observed which shows the convergence trend of the iterative algorithm.

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