Application of Parametric Bootstrap Technique for Damage Detection of a Scaled Reinforced Concrete Bridge

D. SAFI, S. PAKZAD and R. BLUM

ABSTRACT

A growing area of interest in structural health monitoring involves employing sensor networks on major infrastructure to monitor their structural condition and integrity. The use of statistical analysis on the sensor network data is critical in processing the information and establishing frameworks for interpretation and comparison. With enough data, a set of modal parameters can be estimated and used to measure the condition of a structure. Sensor networks in structural engineering applications produce small data sets in many cases. This may be due to events such as an earthquake when the duration of the event is limited to tens of seconds and hence the data sets are small, or immediately after an event when there is an urgency to make statistical inference and decisions with limited data. In the absence of abundant data, many of the proposed statistical algorithms in this field become ineffective. The bootstrap method is a statistical technique that can be used to generate the desired statistics for the estimated parameters. This paper describes this method for indirect detection of structural damage through the measurement and interpretation of white noise vibrations. First, a model of a single span bridge is numerically simulated to demonstrate the detection procedure. This is followed by performing similar analysis on real data from a scaled reinforced concrete bridge test at the University of Nevada, Reno. The contribution of this work is the integration of system identification with the bootstrap technique to find both modal parameters and confidence intervals. This approach provides an effective basis to identify changes in the modal properties and the structural integrity of a bridge.

INDEX TERMS

Parametric Bootstrap, Modal Identification, Model Calibration, Reinforced Concrete Bridge, Sensor Networks

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INTRODUCTION

A sensor network consists of spatially distributed sensor nodes, which cooperatively monitor physical or environmental conditions at different locations and aggregate the data to obtain information about the monitored system. Sensor networks hold great promise in a number of applications. One particular application of interest involves monitoring vibrations to judge structural integrity in infrastructure [1], [2].

In many structural monitoring and maintenance procedures, direct damage detection by visual inspection is required. This is often prohibitive in costs. With sensor-based monitoring, diagnosis information can be extracted indirectly from features of the measured response taken under usual operating conditions.

This paper describes a method for indirect detection of structural damage through the sensor measurement and interpretation of ambient vibrations. First, a model of a single span bridge is numerically simulated to demonstrate the detection procedure. Acceleration data is collected from this model under three cases: no damage, small damage and large damage. The Natural Excitation Technique, NExT [6], and the Eigenvalue Realization Algorithm, ERA [3], is used to determine the modal properties of the structure. A statistical analysis is performed on the modal parameters in order to establish a framework for interpretation and comparison between the three cases.

Since the data sets in structural monitoring applications can be small in size due to the limited duration of an event or small memory size, conventional statistical methods cannot be effectively used to establish confidence measures for the estimates. Statistical measures for the desired modal parameters can still be extracted in such cases using the bootstrap technique. The parametric bootstrap is used to generate statistical properties of the modes, including their confidence intervals. The estimated modal parameters from the three cases are compared in a statistical framework to detect change. This is followed by performing the same analysis on real data from a scaled reinforced concrete bridge test at the University of Nevada, Reno.

The contribution of this paper is the integration of system identification with statistical techniques to find both modal parameters and confidence intervals. This approach provides an effective basis to identify changes in the modal properties and the structural integrity of a bridge.

VIBRATION MONITORING

Many models and approaches have been proposed for structural identification. One particular approach, widely used in modal realization of output-only systems, is based on integration of the Eigensystem Realization Algorithm (ERA) and the Natural Excitation Technique (NExT) [6], and is briefly explained in this section.

Eigensystem Realization Algorithm

The Eigensystem Realization Algorithm, or ERA [3], is a technique used for modal parameter identification. The algorithm consists of two major parts, basic formulation of the minimum-order realization and modal parameter identification. The Eigensystem Realization Algorithm is the most widely used algorithm for modal identification. The details of this algorithm are shown in [3] and [4]. [5] and [6] present examples of application of the algorithm for modal identification of bridges.

Natural Excitation Technique

The ERA algorithm is designed to work with Markov Parameters (impulse responses of the system). When the ambient vibration response is measured instead of the impulse response, an ambient vibration system identification method called Natural Excitation Technique (NExT) can be used to generate the Markov Parameters([7], [8], and [9]). The NExT method involves applying time domain curve fitting algorithms (like ERA) to cross-correlation functions between various response signals to estimate the resonant frequencies and modal damping. The cross-correlation function between two response measurements made on an ambiently excited structure is shown to have the same form as the systems impulse response function. Therefore, instead of using impulse response functions to perform ERA, one can use cross-correlation functions.

Choosing the Correct Modes

The NExT/ERA algorithm can give many modes/natural frequencies. Some of these are true modes and some are the computational spurious modes due to noise or other errors. Modes with large damping ratios can be dropped, as a damping ratio outside an acceptable range (0.001-5) is an indication of false modes. Along with this, in order to determine the correct modes, a stabilization diagram was used. This stabilization diagram shows the minimum model order that provides enough resolution to consistently identify structural modes. The spurious computational modes do not repeat consistently, but true structural modes stabilize as the order increases. With these two methods true modes can be discovered.

PARAMETRIC BOOTSTRAP TECHNIQUE

In order to evaluate the accuracy of the ERA/NExT results, it is essential to derive statistical characteristics of the estimates. One method of doing this is to use resampling.

Definition 1: **Resampling** is a method or class of methods where the accuracy and sampling distribution of a statistic can be estimated or approximated by obtaining samples from a given data set.

This section introduces the popular parametric bootstrap resampling method. Parametric bootstrap [10] is a technique which combines measured sensor data with random simulation to generate desired statistics (in this case statistics of the modal parameters).

Bootstrapping is used to simulate many alternative versions of a single statistic that would ordinarily be calculated from one sample. Therefore, it is possible to derive an estimate of the distribution of the statistic and a measure of its variability.

Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed (iid) random variables from unknown distribution G. Let μ be a statistic that estimates a parameter of the distribution G.

Definition 2: A **bootstrap sample** $\hat{x} = (\hat{x}_1, \ldots, \hat{x}_n)$ is a random sample of size n where each $\hat{x}_i$ is obtained with probability $1/n$ by drawing with replacement from the original sample $x = (x_1, \ldots, x_n)$.

For example, assume a random sample x of size $n = 6$ is drawn from an estimated distribution G:

$$x = (x_1, x_2, x_3, x_4, x_5, x_6) \quad (1)$$

Then one possible bootstrap sample $\hat{x}$ is:

$$\hat{x} = (x_4, x_1, x_6, x_2, x_4, x_5) \quad (2)$$

Confidence Intervals using Bootstrap

After obtaining an estimate for μ, the next item of interest is finding a confidence interval around the estimate. The confidence interval (CI) has this form:

CI = (Point Estimate) $\pm$ (Margin of Error)
= (Point Estimate) $\pm$ (Standard Score) $\times$ (Standard Error) (4)

The sampling distribution of the sample mean is approximately normal, even if the distribution of the population from which the sample is taken is not normal. Therefore, the standard score is the z-score. Let α be a real number between 0 and 1. Typically small values such as 0.01 and 0.05 are used. A $(1¡\alpha) \times 100\%$ confidence interval is defined as:

$$\hat{\hat{\mu}} \pm (z \times \hat{e}) = 1 - \alpha \quad 5)$$

where $\hat{\hat{\mu}}$ is the mean of the distribution of means, $\hat{e}$ is the standard error of the distribution, and z is the z-score for the particular confidence interval of interest.

Hence, to get a confidence interval, it is necessary only to multiply the standard error by the z-score of the points in the normal distribution that exclude $\alpha/2$ of the distribution on either end (two-tailed). For example, a 95% confidence interval $(\alpha = 0.05)$ is defined by a z-score of 1.96.

SIMULATION, TEST SETUP AND RESULTS

Simulated Bridge

In this section, a model of a single span bridge is numerically simulated to demonstrate the identification procedure. Acceleration data is collected from this model under three cases: no damage, small damage and large damage. The Natural Excitation Technique, NExT, and the Eigensystem Realization Algorithm, ERA, was used to determine the modal properties of the structure. Since the bridge model was numerically simulated "exact" solutions (modal parameters) are directly extracted by solving the force equations. These values are compared to the results of the NExT/ERA algorithm.

Table I presents a comparison of the natural frequencies between the NExT/ERA algorithm and their exact values. Five natural frequencies (for each of three cases) are found using the simulated data along with their 95% confidence intervals (lower and upper bounds). The table shows that the exact values fall within the 95% confidence bound for all of the modes in each case.

TABLE I. COMPARISON OF SIMULATION AND EXPECTED NATURAL FREQUENCIES

	Undamaged case		Small damage case		Large damage case	
	Simulation results	Exact Values	Simulation results	Exact Values	Simulation results	Exact Values
Lower CB	4.3056	4.4049	4.2063	4.3347	3.9648	3.9843
mean	4.5609		4.4172		4.0801	
Upper CB	4.8162		4.6281		4.1954	
Lower CB	8.5288	8.6496	8.1000	8.5036	7.8158	7.7269
mean	8.7432		8.5123		8.1969	
Upper CB	8.9575		8.9246		8.5781	
Lower CB	10.7344	10.9274	10.0808	10.6066	9.7667	9.8889
mean	11.1547		10.7639		10.4085	
Upper CB	11.5751		11.4471		11.0503	
Lower CB	13.9304	14.0723	13.3445	14.0407	13.6388	13.9652
mean	14.7315		14.3520		13.9836	
Upper CB	15.5326		15.3596		14.3283	
Lower CB	24.5370	25.7492	22.1654	23.8007	19.8079	20.4867
mean	25.2389		23.2043		20.2361	
Upper CB	25.9408		24.2431		20.6643	

Scaled Reinforced Concrete Bridge

The next step is to implement the ERA/NExT algorithm along with parametric bootstrapping technique to find the modal properties of a scaled bridge. A series of shaking table tests were performed on a 1/4-scale, reinforced concrete bridge with two spans supported by two column bents at the University of Nevada, Reno. The design, instrumentation, test protocol, and bridge response are described by Johnson, et al. [11].

The shaking table tests consisted of 23 ground motions that were applied in the transverse direction. This included 14 low-level tests prior to yielding and 9 high-level tests of increasing amplitude up to a peak ground acceleration (PGA) exceeding 2.0g.

At drift ratios approaching 8%, substantial damage was observed in the reinforced concrete columns. White-noise tests were conducted throughout the test procedure to monitor the progression of damage. The bridge was instrumented with accelerometers at each of the tables as well as at locations along the deck. From these white-noise tests three were selected to demonstrate the effectiveness of the bootstrap technique in detecting damage. The three datasets/cases that are considered are the WN0001 (undamaged), WN1112A (small damage), and WN1920 (large damage). This structure has three modes with frequencies below 15 Hz, which change due to damage.

The confidence intervals for the three natural frequencies are presented in the following figures. Figure 1 shows the 95% confidence interval for the natural frequency of the undamaged case. Figures 3 and 4 show the confidence intervals of the natural frequencies for the small damage and large damage cases respectively. The severity of damage can be qualitatively determined by examining the confidence intervals of the natural frequencies. In particular, the natural frequencies decrease with additional damage. This is shown in figure 4 for the three natural frequencies.

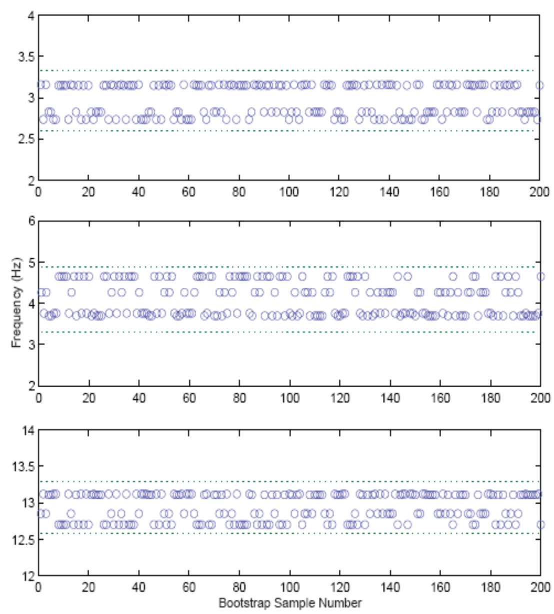
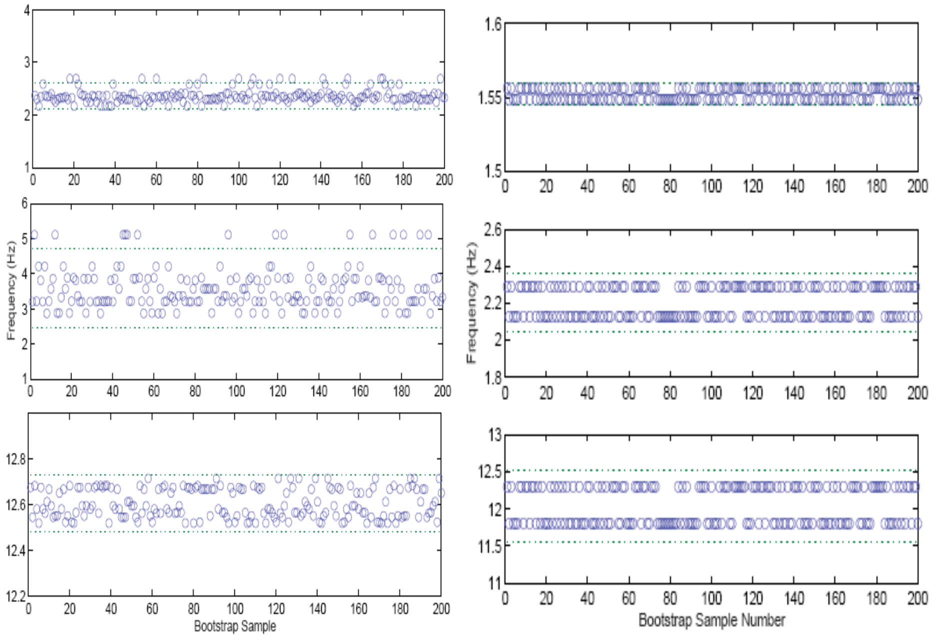


Figure 1: Confidence Intervals for the undamaged case (WN0001)

WN1112A case: WN1920 case:



Figures 2, 3: Confidence Intervals for the small damage case (WN1112A) and the large damage case (WN1920)

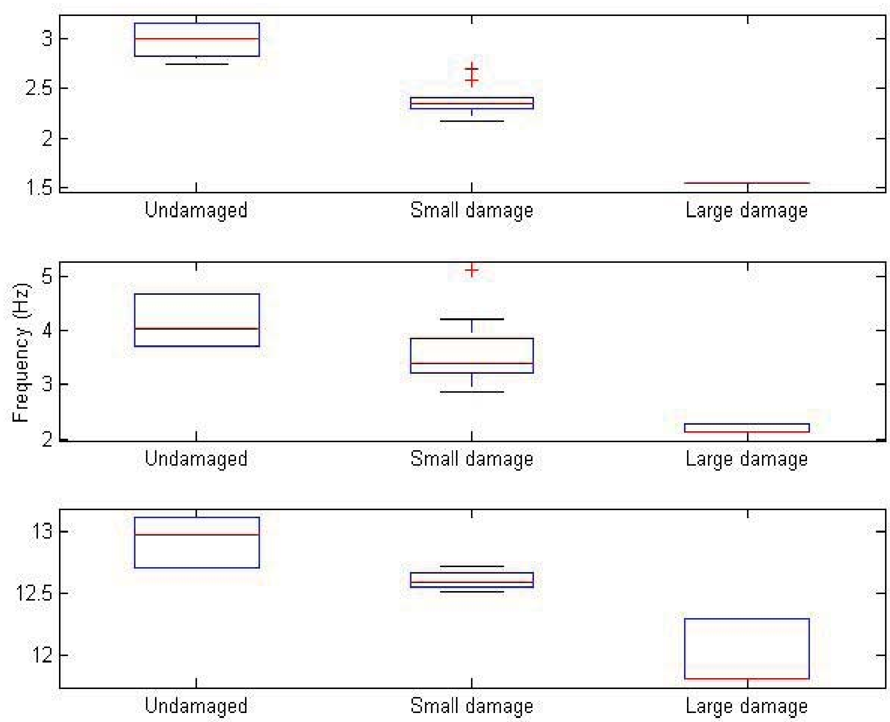


Figure 4: Mean and confidence intervals for each case for the three natural frequencies

CONCLUSION

In this paper a method was described for indirect detection of structural damage through the measurement and interpretation of ambient vibrations. A model of a single span bridge was numerically simulated to demonstrate the detection procedure. Acceleration data was collected from this model under three cases: no damage, small damage and large damage. The Natural Excitation Technique, NExT, and the Eigenvalue Realization Algorithm, ERA, was used to determine the modal properties of the structure. A statistical analysis was performed on the modal parameters in order to establish a framework for interpretation and comparison between the three cases and detect damage.

The parametric bootstrap was used to generate statistical properties of the modes, including their confidence intervals. The estimated modal parameters from the three cases are compared in a statistical framework to detect change.

This was followed by performing the same analysis on real data from a scaled reinforced concrete bridge test at the University of Nevada, Reno. The results showed that the integration of system identification with statistical techniques which finds modal parameters along with confidence intervals provide an effective basis to identify changes in the modal properties and structural integrity of a bridge.

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