Chapter 23
A Parameter Optimization for Mode Shapes Estimation Using Kriging Interpolation

Minwoo Chang and Shamim N. Pakzad

Abstract A parametric study of Kriging interpolation for Optimal Sensor Placement (OSP) is presented in this paper. A Kriging model uses geostatistical information to interpolate and extrapolate the values for unobserved locations with a weighted sum of known neighbors. The accuracy of mode shape estimates is evaluated by Modal Assurance Criteria (MAC), compared to the target mode shapes. The performance of OSP is enhanced by the Kriging results. For the quality estimation of mode shape, a parametric study is conducted in this paper. The Kriging model is composed of linear regression model with random error which is assumed as a realization of a stochastic process. A Gaussian function is used to characterize the covariance function between two random errors in terms of their relative distance. Three parameters are involved to define covariance function: regression model order and two amplification parameters. The parameter optimization approach aims at OSP solution with the minimum number of sensors. The effect of parameters is evaluated using numerically simulated harmonic modes, and modes from Northampton Street Bridge (NSB). Modified Variance (MV) is used to rank the signal strength at candidate sensing locations. The results show that the accuracy of estimated mode shapes is dependent on the eigenvalue of covariance matrix and the number of sensors can be minimized when the Kriging model is optimally designed.

Keywords Optimal sensor placement • Kriging • Modal assurance criteria • Modified variance • Wireless sensor network

23.1 Introduction

Optimal Sensor Placement (OSP) is a common issue for all engineering systems such as mechanical, aerospace, and structural, which require the posterior monitoring to maintain their performance [1–3]. From a set of observations, the indices can be extracted to quantify the systems' performance and modal parameters can be updated to reflect the behavior of existing systems. In particular, the effect of OSP is significant for structural systems, since the scale of civil infrastructure is much larger than other engineering problems. The traditional sensor networks, represented by wired sensors, are costly and physically limited to properly place them on large scale structures. Although the recent development of Wireless Sensor Networks (WSN) have facilitated the possibility of vibration monitoring with densely located sensors [4–6], the costly process for data transmission is inevitable and an optimum number of sensors is a more effective solution for practical Structural Health Monitoring (SHM).

The main objective of OSP is to formulate the sensor configuration to detect significant changes representing structural damage based on the evaluation of the signal strength at sensing locations [3, 7]. Additionally, the use of OSP reduces the cost by eliminating collecting large volume of redundant sensor data as well as managing the number of sensors. Effective Independence (EI) method [1], one of the widely used OSP techniques, examines the error of unbiased estimator using target mode shapes. For a variation of EI, modal frequency and modal mass are additionally used to quantify signal strength in EI-Driving Point Residue (DPR) [8] and Kinetic Energy [9] methods, respectively. Alternatively, the methods to search principal component in target mode shape matrix have been developed using correlation [10] and covariance [11] of mode

M. Chang (✉) • S.N. Pakzad
Department of Civil and Environmental Engineering, Lehigh University, 117 ATLSS Drive, Bethlehem, PA 18015, USA
e-mail: mic307@Lehigh.edu; pakzad@Lehigh.edu

G. Foss and C. Niezrecki (eds.), *Special Topics in Structural Dynamics, Volume 6: Proceedings of the 32nd IMAC, A Conference and Exposition on Structural Dynamics, 2014*, Conference Proceedings of the Society for Experimental Mechanics Series,
DOI 10.1007/978-3-319-04729-4_23,

shape matrix. In order to expand the applicability of previously developed variance method [11], Modified Variance (MV) method is introduced in which the computational time to define the best sensor configuration is much reduced [7].

In order to provide an automatic guideline for optimal sensor configuration, a framework, based on Modal Assurance Criteria (MAC) comparison, is introduced and serves to decide the minimum number of sensors on most informative locations [7]. The estimated mode shapes using interpolation/extrapolation techniques are compared to the target mode shapes using MAC to analyze the compatibility of a chosen sensor configuration. The polynomial functions are facilitated in many engineering problems and have shown reliable estimation results [12]; however, the boundary conditions should be known for satisfying the continuity of piecewise components. Alternatively, Kriging method, which is a geostatistical estimator of the unknown values at unobserved locations, can be used with only geometric information at candidate sensor locations to resolve the need for boundary conditions [13]. In order to define Kriging model, the error of mode shape function is assumed as random and its covariance is mathematically designed with several parameters.

This paper aims to investigate Kriging technique and to observe the effect of associated parameters for accurate mode shape estimation. Two examples are used to demonstrate optimal sensor configurations. EI and MV methods are investigated to quantify the signal strength at candidate sensor locations. The results of OSP are used to estimate the mode shapes for all candidate sensor locations.

23.2 Framework for Optimal Sensor Configuration

A framework to design optimal sensor configuration is introduced in [7]. The framework is comprised of two sub-tasks: (1) estimation of signal strength at each sensing node and (2) MAC comparison between target modes and estimated modes from a set of optimal sensor configuration. The target mode information is used to define the best sensor locations depending on the number of sensors. For a particular set of sensor configuration, the mode shapes are estimated for the unobserved locations using interpolation/extrapolation techniques. The MAC between exact mode shape and estimated from a set of optimal sensors is calculated. The best sensor configuration is defined when the minimum MAC amongst all target modes is beyond the threshold; otherwise the next sensor configuration with an additional sensor is selected and this procedure is repeated until the constraint for the MAC threshold is satisfied.

23.2.1 Optimal Sensor Placement (OSP) Methods

In this study, two OSP methods, Effective Independence (EI) [1] and Modified Variance (MV) [7], are investigated to quantify the signal strength at candidate sensor locations depending on the target modes. These two methods are convenient to implement since the methods require only the target mode shape information and are computationally inexpensive. The following is a brief description for each method.

Effective Influence (EI): An unbiased estimator of modal contribution parameter is defined as a function of target modes. The numeric deployment of error covariance between the exact modal contribution parameter and the unbiased estimator is the inverse of Fisher information matrix which is the expected value of the observed information. The Effective Independence Distribution (EID) vector is introduced to quantify the contribution of the candidate sensor locations. A sensor location noted with a lowest index of EID is discarded from the candidate locations and this procedure is repeated to determine the priority of sensor locations.

Modified Variance (MV): The covariance of target mode shape matrix is used to quantify the signal strength of candidate sensor locations. In order to prevent the irregularity in covariance due to sign convention and to increase independency of modal information at nodes, the target mode shape matrix is transformed by attaching the negative of mode shape matrix to the original. The unbiased estimator of mode shape function at unobserved locations is defined as a function of covariance matrix. The error of this estimator can be minimized when the determinant of covariance of observed locations is maximized. For the practical implementations, a *pc* index is introduced, considering the dispersion of off-diagonal components. Similar to EI method, a sensor location with lowest signal strength is tossed from the candidate sensor locations.

23.2.2 Kriging

Kriging is a geostatistical estimator to infer the random values at unobserved locations where the signal response can be estimated using weighted sum of known values at neighbors [14]. The shape function is initially estimated by the sum of a linear regression model and random error as:

$$y_i = p(x_i)^T a + z(x_i) \tag{23.1}$$

In Eq. 23.1, y_i is a modal ordinate at x_i; $p(x_i)$ is a set of non-linear bases, for example $p(x_i) = \left[1 \; x_i \; x_i{}^2\right]$ for a quadratic basis; a denotes a coefficient vector which minimizes the random error $z(x_i)$. The covariance of the error is defined by the correlation between two sensor locations, with high correlation for closely spaced sensors. The correlation function is frequently expressed by using Gaussian functions in which two parameters are involved.

$$R\left(x_i, x_j\right) = \alpha \exp\left(-\theta r_{ij}{}^2\right) \tag{23.2}$$

In Eq. 23.2, α and θ are the amplification parameters and r_{ij} is the physical distance between x_i and x_j. Considering that the a is canceled out for the mode shape estimation, the order of the basis and correlation parameter θ, need to be designed for Kriging model. The details to estimate mode shapes are presented in [13, 15].

23.3 Case Studies

Two examples are used to verify the effectiveness of the framework to formulate the sensor configuration and to observe the effect of two parameters. In a numerically simulated shear building model, the effect of Kriging parameters is investigated and the best sensor configuration is found. Amongst the parameters, the effect of the order of basis and correlation parameter θ are considered since α does not affect the mode shapes estimation. In the second example, the modal parameters estimated from the response from the Northampton Street Bridge (NSB) are used to optimize sensor configuration depending on the number of target modes and applied OSP methods.

23.3.1 Shear Building Model

A 19-DOF simply supported beam is used to simulate modal parameters as shown in Fig. 23.1 (first five vibration modes). Kriging can be used effectively to interpolate the mode shapes since it does not require the continuity at the top of the shear building model. EI and MV methods are applied to determine the sensor location priority chart. In order to define the number of sensors, MAC of 0.95 for all target modes are set as a constraint.

For harmonic mode shapes, the minimum number of sensors is determined, which is equal to the number of target modes. For example, the changes of the optimal number of sensors and corresponding determinant of correlation matrix are illustrated in Fig. 23.2a. The constant basis for correlation is used. In this figure, the optimal number of sensors decreases

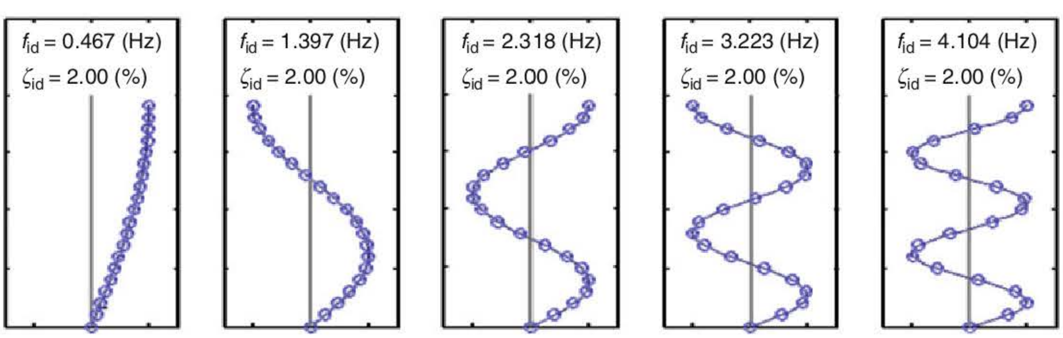


Fig. 23.1 First five mode shapes in 19DOF model with natural frequency and damping ratio

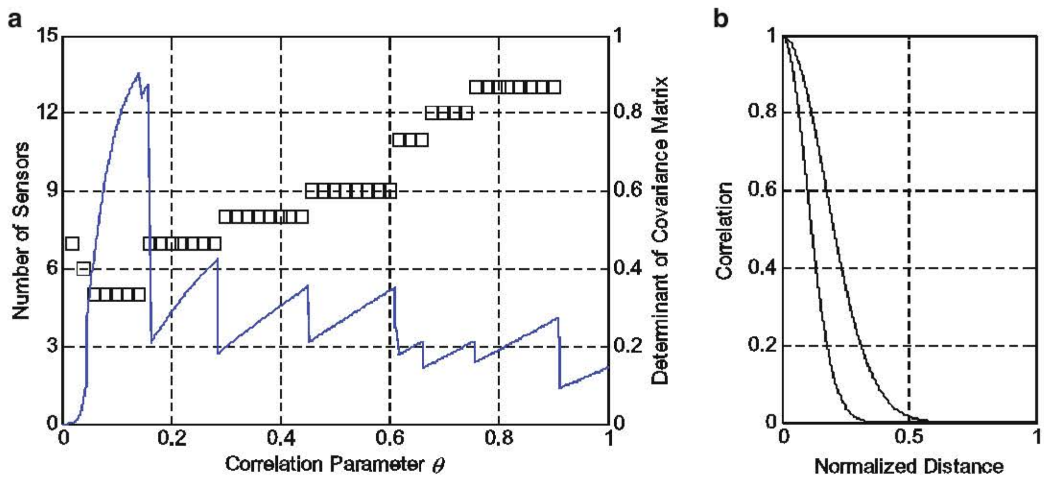


Fig. 23.2 Effect of correlation parameter to minimize number of sensors when five modes are targeted. (**a**) Number of sensors and determinant of target modes (**b**) Feasible range of correlation functions

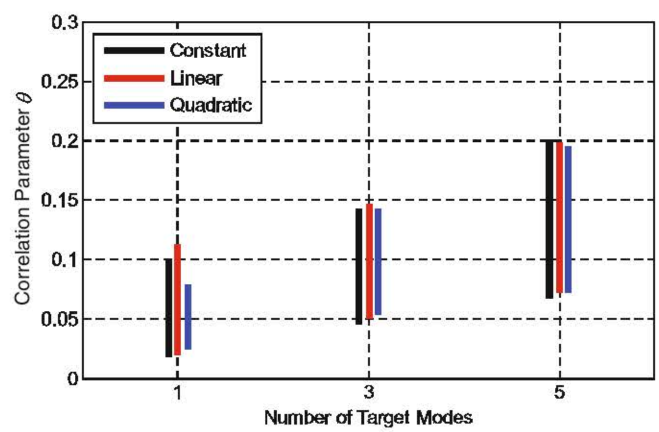


Fig. 23.3 Ranges of optimal coefficient parameter for three orders of basis

rapidly for $\theta < 0.46$ and increases as the correlation parameter increases. In particular, the feasible range of θ is determined between (0.046 and 0.142) for which five sensors are determined as an optimal solution. The determinant of correlation function is discontinuous when the sensor configuration changes. The feasible range of correlation function is plotted in Fig. 23.2b, indicating the high correlation for closely located sensors and the random errors are almost independent when the normalized distance between two sensor locations is larger than the half of height of the structure.

Further investigation is conducted to evaluate the effect of the order of basis. The basis for the one dimensional problem is determined as power series of candidate sensor location; for example, [1] for constant basis, [1 x_i] for linear basis, and [1 x_i x_i^2] for quadratic basis can be used. Although the increase of order requires additional computational cost, it does not necessarily improve the quality of estimated mode shape and reduce the number of sensors. Figure 23.3 shows the feasible ranges of correlation parameter θ when the numbers of target modes are varied. For all cases, the optimal numbers of sensors are the same with the number of target modes. The higher orders of basis are excluded in this figure since the optimal numbers of sensors are larger than the optimal number of target modes. In general, the linear basis slightly expands the feasible range of correlation parameter and the high order shows negative effect to search the best sensor configuration.

23.3.2 Northampton Steel Bridge

The ambient vibration response of Northampton Street Bridge (NSB) was measured using Wireless Sensor Network (WSN) and used to identify modal parameters in vertical responses [16]. A total of 21 wireless sensor units (18 on the North Side and three on the South Side) measure the ambient vibration of the bridge (Fig. 23.4). Structural Modal Identification Toolsuite (SMIT) [17] is used to identify seven structural modes under 10 Hz frequency range. The modal ordinates for sensors on south sides are used to distinguish the type of modes (vertical and torsional).

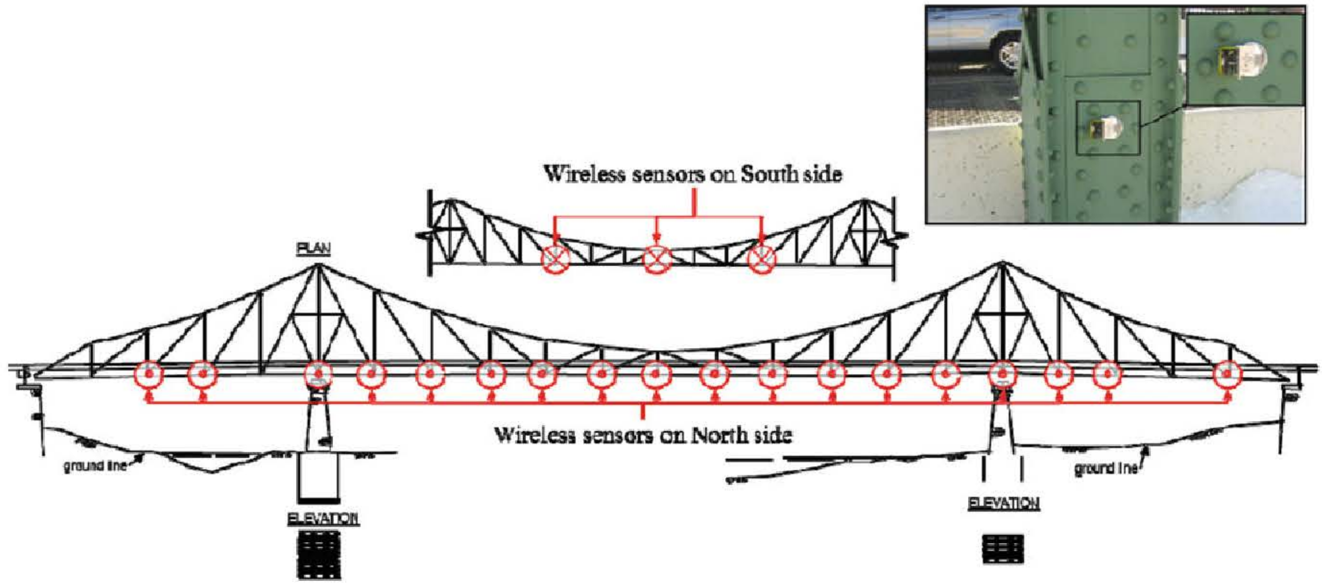
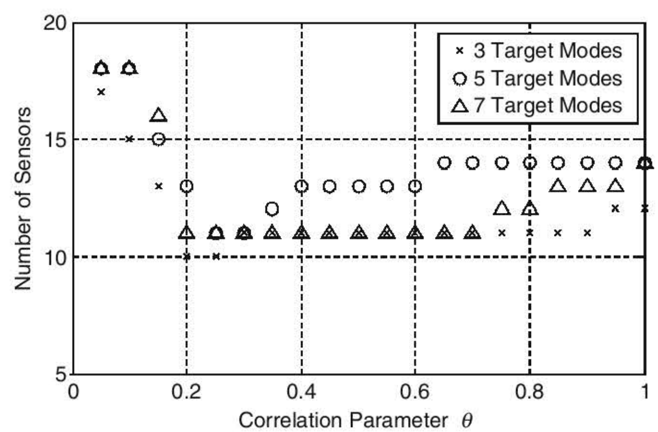


Fig. 23.4 Wireless sensor network in Northampton Street Bridge

Fig. 23.5 Optimal numbers of sensors versus correlation parameter



The optimal numbers of sensors to observe three, five, and seven target modes are plotted versus correlation parameter when MV method is employed (Fig. 23.5). Similar to the shear structure model, the number of sensors decreases rapidly first; then increases as the correlation parameter increases. For this example, very small range of correlation parameter is available for the optimal solution. In general, the numbers of sensors required to observe several sets of target modes are similar due to the complex mode shapes caused by two piers in the middle of bridge and the insufficient information of low number of target modes.

The optimal numbers of sensors for EI and MV methods are plotted (Fig. 23.6). For both methods, the required sensors are similar when the targeted modes are larger than four. As reported in [7], the insufficient information in target mode shape matrix results in the false estimation of signal strength and requires more sensors even though the fewer number of modes are targeted. EI and MV methods show similar performance to optimize the number of sensors regardless of the number of target modes.

In order to observe the way to determine sensor locations for each OSP method, the step by step procedures are tabulated when seven modes are targeted (Table 23.1). EI method detects two highest signal strengths at the end of the bridge while MV method evaluates highest signal strength at the middle. The sensors are almost uniformly spaced when the seventh sensor location, which is the minimum number of sensors when the rank of seven for target mode shape matrix is chosen. Since it fails to estimate MAC of 0.95, extra sensor locations are added for the optimal sensor configurations. For this particular example, the performance of MV (11 sensors) is better than EI (12 sensors). The sensor configurations become the same when 12 sensor locations are selected.

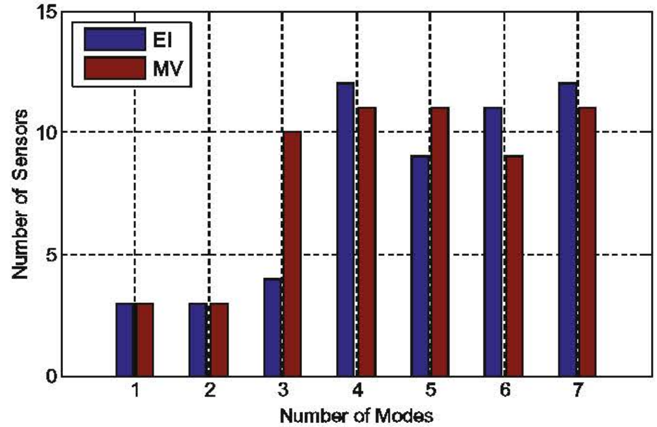
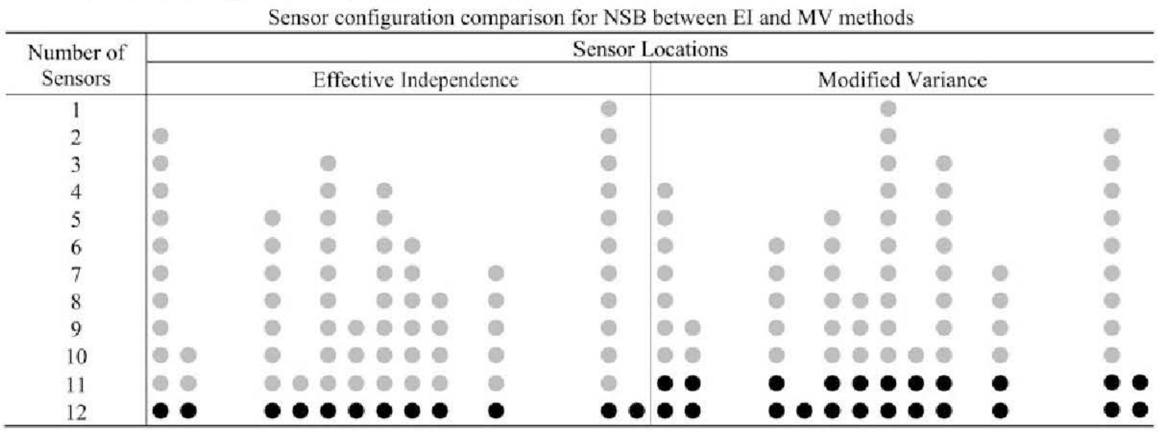


Fig. 23.6 Optimal numbers of sensors versus number of target modes for EI and MV methods

Table 23.1 Sensor configuration comparison for NSB between EI and MV methods



23.4 Conclusion

In this study, the effects of Kriging parameters, the order of basis for mode shape estimator and the correlation parameter for random error are investigated. Kriging model can be used to estimate the mode shape effectively since it utilizes the geometry information whereas other methods such as Spline require continuity condition at the boundaries.

Two applications are used to observe the effect of the Kriging parameters. A 19-DOF shear structure model shows the effect of the order of basis function is insignificant and low order is recommended to reduce the computational cost. The correlation parameter needs to be carefully selected. The feasible range for the correlation parameter always exists, which requires further research for the automatic search for the best sensor configuration. The modal parameters for NSB from a set of ambient vibration response are used for optimal sensor configuration. In general, the feasible range of correlation parameter is shorter than shear structure. Due to the complexity in mode shapes and insufficient information in target mode matrix, many sensors are required. The EI and MV methods are used to quantify the signal strength. Both show similar sensor configuration with almost uniform spacing.

Acknowledgements This research was partially supported by the National Science Foundation under grant CMMI-0926898 by Sensors and Sensing Systems program, and by a grant from the Commonwealth of Pennsylvania, Department of Community and Economic Development, through the Pennsylvania Infrastructure Technology Alliance (PITA).

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