

A Compressed Sensing Approach in Structural Damage Identification

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ABSTRACT

Accurate detection of time and location of damage in its early stages has motivated the researchers to develop several damage diagnosis methods in recent decades. With current improvements in monitoring hardware, many of the previously developed Structural Health Monitoring (SHM) methods encounter the BIG DATA problem in processing the data collected through dense sensor networks. Therefore, it is vital to improve the efficiency and scalability of today's SHM procedures parallel to data acquisition enhancements. Toward this end, this paper presents a data-driven damage detection methodology based on compressed sensing techniques. The objective of the paper is accurate damage localization in a structural component instrumented with a dense sensor network, by processing data only from a subset of sensors. In this method, first a set of sensors from the network are randomly sampled. Measurements from these sampled sensors are processed to extract damage sensitive features. These features undergo statistical change point analysis to establish a new boundary for a local search of damage location. As the local search proceeds, probability of the damage location is estimated through a Bayesian procedure with a bivariate Gaussian likelihood model. The decision boundary and the posterior probability of the damage location are updated as new sensors are added to processing subset and more information about location of damage becomes available. This procedure is continued until enough evidence is collected to infer about damage location. Performance of this method is evaluated using a finite element model of a cracked gusset plate connection. Pre- and post-damage strain distributions in the plate are used for damage diagnosis.

INTRODUCTION

Accurate and timely detection of damage in structures and infrastructure systems is the primary goal of Structural Health Monitoring (SHM) procedures. Several methodologies have been developed over recent decades in response to this goal [1].

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These methods are categorized based on the underlying models used to train the data to identify potential anomalies in the structure (i.e. damage). In one category of these techniques monitoring data are used to update uncertain parameters of a physics-based model of the structure and change in the updated parameters signify the damage [2-4]. In the other category, time series and signal processing techniques are directly employed on the measured signals to extract damage sensitive features and monitor them for potential changes [5-8]. Although several techniques have been proposed to improve the efficiency of model-based SHM techniques [9-10], data-driven methods seem more promising for rapid - and ultimately real-time - detection of structural damage.

Rapid improvements of monitoring hardware enable the measurement of structural response with high resolution in time and space. With such improvements, many of the previously developed SHM methods encounter the BIG DATA problem in processing the data collected through such dense sensor networks [11]. Therefore, it is vital to improve the efficiency and scalability of the SHM procedures. Toward this goal, in recent years data compression techniques have attracted attention of SHM research community. One of such methods is compressed sensing where structures' response is transmitted or processed in a compressed form. Structural damage detection or modal quantity estimation will be then performed using the compressed sensing coefficients or reconstructed signals from them [12-14].

This paper presents a damage localization technique with a compressed sensing approach. The data compression in the proposed process is done through analyzing a subset of sensors from a dense sensor array to accurately detect the timing and location of structural damage. Previously, Yao et al. [15] proposed a compressive sensing scheme damage detection method based on spatial correlation of random samples and Ant Colony optimization. In the present paper, damage detection starts with selecting a subset from entire monitoring network. Data from these sensors are processed for features extraction and change point analysis. When the change point analysis signifies a potential time and location for damage, neighborhood of the suspect location is investigated further in a local sampling step. A recursive Bayesian estimation procedure is also adopted in order to iteratively update the probability of damage location as data from more sensors are considered for processing. This procedure is terminated when damage is localized with a certain probability. The following sections of the paper describe this damage localization methodology in details. Performance of this technique is also shown using a finite element (FE) model of a steel gusset plate.

COMPRESSED DAMAGE DETECTION AND LOCALIZATION

The data-driven damage detection methodology proposed here consists of iterative global and local sampling steps from a dense sensor network. With a uniform prior probability for the location of damage, the global sampling step starts by taking samples uniformly from the entire sensor network. This iterative global sampling ensures high reliability in finding a proper start point and establishing an initial local search boundary. Data from the sampled sensors are processed for feature extraction and statistical testing using change detection methods. As test statistics from sampled sensor locations cross the specified change threshold, likelihood of the damage

location is calculated, and is used to obtain the posterior probability of the damage location. The local sampling steps start with taking samples inside a smaller window centered on the location with maximum posterior probability. The steps of local sampling, feature extraction, change point analysis, likelihood and posterior probability estimation are repeated until damage is localized with a pre-specified probability. The local sampling window is also updated if a new sampled point reveals the highest change statistics over the current search window. Figure 1 shows the details of the proposed method in a flowchart.

APPLICATION OF PROPOSED COMPRESSED DAMAGE DETECTION METHOD ON A STEEL GUSSET PLATE

Accuracy and robustness of the proposed analytical compressive damage diagnosis framework is evaluated by FE simulations of damage and undamaged structural connections used to generate strain data.

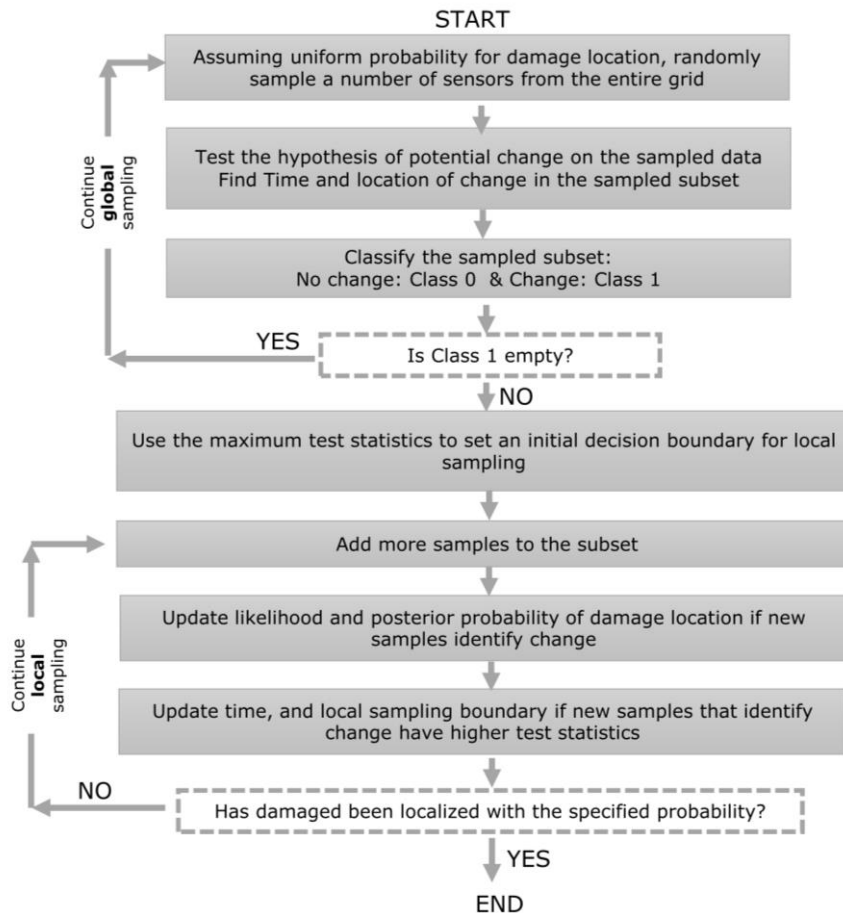


Figure 1. Flowchart of the proposed compressed damage localization algorithm

Figure 2 shows the simulated two-way gusset plate connection used for numerical validation in this paper. The assembled connection is 52 inches and undergoes a 50 kips axial tensile load. It should be noted that the gusset plate is designed to withstand up to 100 kips of axial tensile force. The simulated damage is a one inch long cut in

the free section of the gusset plate. Strain filed at the middle section of the gusset plate before and after damage is used to simulate the test data. In both damaged and undamaged cases, white Gaussian noise is added to the data to create a more realistic monitoring scenario and generate 30 sets of strain data for each structure's health condition.

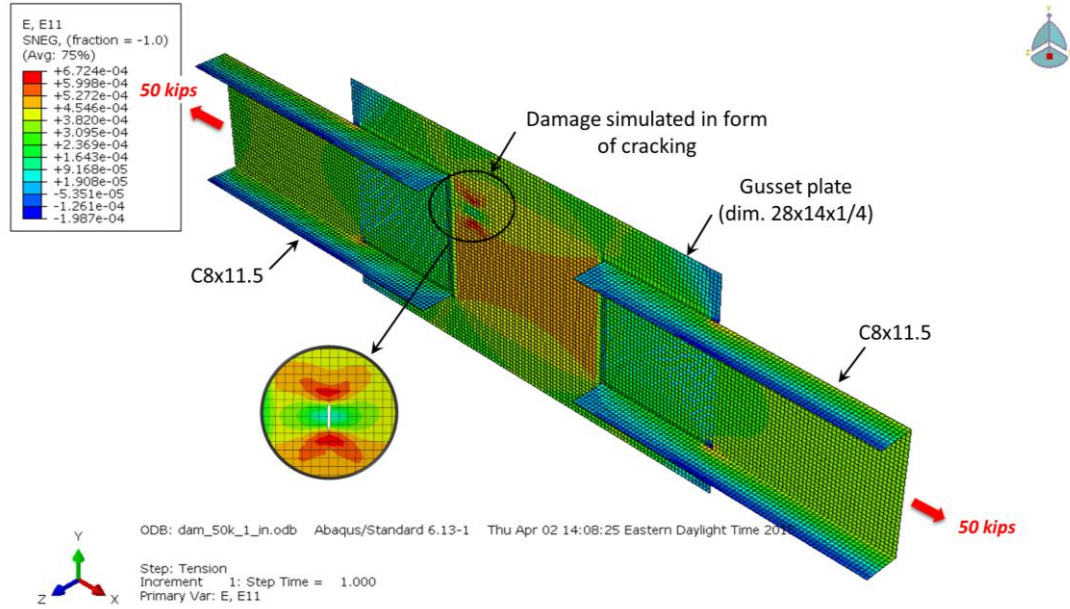


Figure 2. Simulated gusset plate connection under axial loading

Damage Features Extraction

The damage feature used in this study is a dimensionless scalar feature based on the relative change in the strain at neighboring grid nodes. Since direction of the potential cracking is not known in real damage cases, this feature establishes a relationship between strain at every node of the FE mesh and those from points closest to that node in the two orthogonal directions. Eq. 1 shows this damage feature. In this equation $\epsilon_{i,j}$ denotes strain at a node with coordinate (i,j) .

$$DF_{i,j} = \left| \frac{\epsilon_{i,j}}{\epsilon_{i+2,j}} \right| + \left| \frac{\epsilon_{i+2,j}}{\epsilon_{i,j}} \right| + \left| \frac{\epsilon_{i,j}}{\epsilon_{i,j+2}} \right| + \left| \frac{\epsilon_{i,j+2}}{\epsilon_{i,j}} \right| \quad (1)$$

When the gusset plate is intact, each term in Eq.1 is close to unity, since there is no abrupt change of strain between neighboring nodes in the middle section of the gusset plate. When a crack is formed; however, a drastic change in the vicinity of the crack occurs in form of stress reduction along the cracked section and intensified stress around the crack tips. With these changes, the damage features would deviate from their counterparts extracted from the “healthy” state of the structure. Therefore, in order to statistically test the significance of change in these damage features, vectors

of features shown in Eq. 1 from damaged and undamaged FE models are tested to find a statistically significant change in their means.

Change point Analysis

In order to test the change in the damage sensitive features described before, two-sample t-test is used here. This control statistics is based on the Student's t-test and is a common procedure for testing the differences between the means of two samples [16], and has been successfully adopted for data-driven damage detection [17-18]. The statistics of this test has N-2 degrees of freedom (N being the combined length of the two sample vectors) and is given in Eq. 2:

$$t = \frac{\hat{X}_1 - \hat{X}_2}{S_p \sqrt{(1/n_1) + (1/n_2)}} \quad (2)$$

where the variables \hat{X}_1 and \hat{X}_2 are the means, n_1 and n_2 are the sizes of the two samples, and S_p represents their pooled standard deviation. In order to find the timing of damage, in this detection algorithm, the two-sample t-test is applied sequentially through the vectors of damage features extracted from the sampled sensors' data. In effect, each vector of damage features is iteratively split into two segments (N-2 cases for a feature vector with N elements), and change in the means of two splits are tested. Upper and lower control limits for this test are then calculated using the Student's t inverse cumulative distribution function at a certain confidence level and N-2 degrees of freedom. When a vector of test statistics crosses these control limits, significance of the change in the statistics is inferred. As sensors are located closer to the location of damage, their change statistics increase. This is the basis for finding the location of damage. Timing of the damage is identified as the time corresponding to the maximum test statistics over all splits.

Recursive Bayesian Estimation

In the proposed algorithm, a Bayesian estimation framework [19] is adopted to find the probability of damage over the sensor network to stop the sampling process when enough evidence is available for damage localization. This Bayesian estimation process starts with a uniform prior for the entire grid of the FE mesh under investigation. A bivariate Gaussian model is then utilized to find the likelihood of each potential hypothesis regarding damage location with respect to the new processed data. Eq. 3 shows the formulation of this bivariate Gaussian model, where D_k shows the coordinate of the k^{th} detected change point (i.e. $D_k = [x_d \ y_d]_k$) and $H_{x,y}$ represents the hypothesis that damage is located at the coordinate (x,y).

$$L(D_k | H_{x,y}) = \frac{1}{\sqrt{(2\pi)^2 |\Sigma|}} \exp\left(-\frac{(D_k - [x \ y]) \Sigma^{-1} (D_k - [x \ y])^T}{2}\right) \quad (3)$$

Eq. 4 shows the k^{th} iteration in the Bayesian estimation process.

$$Po_k(H_{x,y}) = \frac{\Pr_k(H_{x,y})L(D_k|H_{x,y})}{\sum_x \sum_y \Pr_k(H_{x,y})L(D_k|H_{x,y})} \quad (4)$$

In this equation \Pr_k and Po_k respectively indicate prior and posterior probability of damage for the k^{th} iteration. As stated before, for $k=1$, a uniform prior is used as there is no knowledge about the location of damage prior to observing a significant change in the sampled sensors' data. For $k > 1$, posterior probability estimated in the previous step (i.e. $Po_{k-1}(H_{x,y})$) is recursively used as the prior probability of the damage location.

RESULTS

This section presents the results of the proposed damage detection method applied on the simulate data. Figure 3 shows the results of the damage localization algorithm when simulated noise has a small amplitude; at each node standard deviation of the noise signal is 1% of value of strain. Figure 3(a) shows the results of global and local sampling steps and figure 3(b) shows the probability associated with finding the damage location at the end of the procedure. In this figure true location of damage, as well as its estimated location is shown. This figure also shows that the location of damage is detected by analyzing 8.5% of the entire data set.

Different noise amplitudes are then considered to investigate the robustness of the proposed methodology to the measurement noise. For each noise level, 50 sets of simulation are performed and damage detection procedure is repeated. Table I and II summarize the results of these simulations. Table I shows the results of damage detection case 1, when change point analysis is used to find the location of the damage on the plate, while the time of the damage is assumed to be known. However, Table II summarizes the results for the case 2 where timing and location of damage is updated as more evidence is provided.

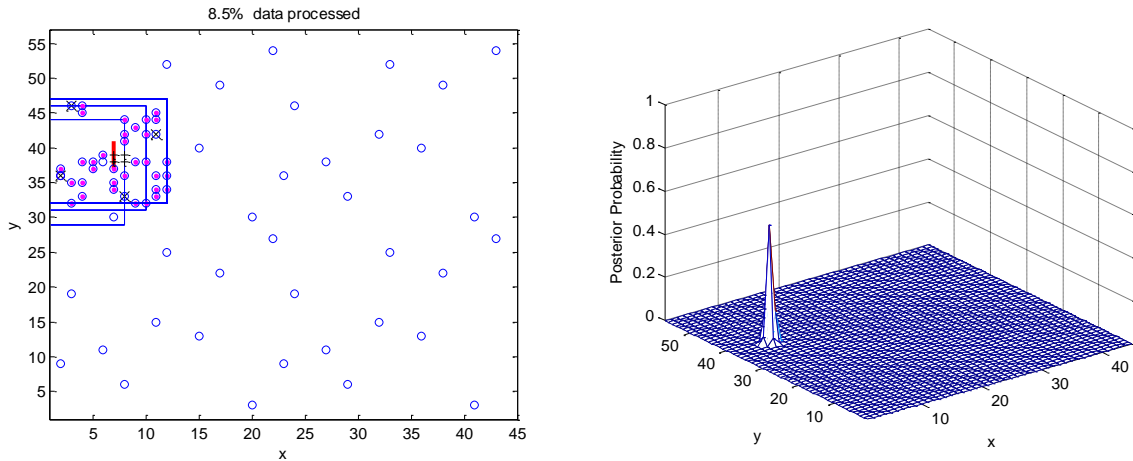


Figure 3. (a) Results of iterative spatial sampling and change point analysis, (b) posterior probability of damage location

TABLE I. DAMGE DETECTION RESULTS: CASE 1

noise level (%)	1	5	10
successful detection (%)	100	98	98
mean error (%) x direction	1.02	0.54	0.09
mean error (%) y direction	1.40	0.93	1.32
data processed(%)	8.24	10.69	13.17

TABLE II. DAMGE DETECTION RESULTS: CASE 2

noise level (%)	1	5	10
successful detection (%)	100	96	86
mean error (%) x direction	0.76	0.56	0.16
mean error (%) y direction	1.40	1.17	1.18
data processed(%)	8.40	10.59	12.65

SUMMARY AND CONCLUSIONS

This paper presents a methodology for compressed damage diagnosis. The main motivation for developing such damage detection methods is to improve the scalability of damage diagnosis frameworks. With rapid advancement in SHM hardware over recent decades, dense contact and non-contact sensor networks are readily used in the monitoring projects, and thus measurements with high resolution in time and space are provided. While higher resolution measurement techniques could be beneficial in accurate structural damage detection, it is important to improve the scalability of damage detection algorithms for processing of SHM BIG DATA. The proposed algorithm in this paper aims to present a method that accurately find the time and location of damage in the structures while a very small subset of sensor nodes are used for processing. The method works on the basis of change point analysis and recursive Bayesian estimation. This algorithm is applied for damage detection in a simulated gusset plate under axial loading. Thirty sets of noisy strain field are generated from undamaged and damaged states of the structure. Different noise amplitudes are then considered to investigate the robustness of the proposed methodology to the measurement noise. For each noise level, 50 sets of simulation were performed and damage detection procedure is repeated. The results show that with processing less than 15% of the monitoring data, this procedure is successful in localizing the damage with low estimation error. As noise increases, the reliability of the method decreases; however, in the worst case -with highest level of simulated noise and with assumption of unknown timing for damage - the procedure is successful in more than 85% of the simulated scenarios.

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